**NCTU Department of Applied Mathematics**

**Qualifying Examination in Discrete Mathematics**

**for the Ph. D. Program** February 2020

<u>**Note**</u>: The proofs and statements must be detailed. When you quote some theorems, please prove them.

1. Build a generating function $g(x)=\sum_{r=0}^{\infty} a_r x^r$ with $a_r=r(r+1)(r+2)$ and evaluate the sum $1\times2\times3+2\times3\times4+\ldots+n(n+1)(n+2)$ for a positive integer n. (20%)

2. Let G be a simple bipartite graph. Give a necessary and sufficient condition for G having a perfect matching, and prove it. (20%)

3. Prove the "maxflow-mincut" theorem (by Ford and Fulkerson, 1956). (20%)

4. Let *n* be a positive integer. Find the chromatic index of complete graph $K_n$. (10%)

5. True or False. (If the statement is true, prove it; if it is false, give a counterexample) (10%×3)
   (a) Let G be a 2-connected of order ≥ 3. Then every three distinct vertices of G are contained in some cycle of G.
   (b) Let T be a tree. If T have a vertex with degree k>2, then T has at least k vertices with degree 1.
   (c) If H is a planar graph, then the chromatic number of H is less than or equal to 6.