Discrete Mathematics

February 2010

<博士班資格考>

1.(10%) Let X and Y be finite sets such that $|X| = n$ and $|Y| = k$. Use Principle of Inclusion and Exclusion to show that the number of the surjections from X to Y is $\sum_{i=0}^{k}(-1)^i \binom{k}{i} (k-i)^n$.

2.(15%) Let $A_1, A_2, \cdots\cdots, A_m$ be subsets of $\{1, 2, \cdots\cdots, n\}$ such that A_i is not a subset of A_j for all $i \neq j$. Show that $m \leq \binom{n}{\lfloor \frac{n}{2} \rfloor}$.

3.(15%) Let n, p be positive integers such that $p \geq 2$ and $n < 2^{p/2}$. Show that there exists a coloring of the edges of K_n in two colors such that no monochromatic K_p exists. i.e., Show that the Ramsey number $N(p, p; 2) \geq 2^{p/2}$.

4.(15%) Suppose that G is a simple graph on n vertices and has more than $\frac{1}{2}n\sqrt{n-1}$ edges. Show that G has girth ≤ 4.

5.(15%) Let r, s, n be positive integers such that $r \leq n$ and $s < n$, and let A be a partial Latin square of order n in which the cell (i,j) is filled if and only if $i \leq r$ and $j \leq s$. Show that A can be extended to a Latin square of order n if and only if $N(i) \geq r + s - n$ for $i = 1, 2, \cdots, n$ where $N(i)$ is the number of i in A.

6.(15%) In a linear space (P,B,I), any two points (in P) are on exactly one line (in B). Let $v = |P|, b = |B|$. Show that if $b \geq 2$, then $b \geq v$.

7. Let G be a simple planar graph.
(1) (7%) Show that G contains a vertex of degree at most 5.
(2) (8%) Show that $\chi(G) \leq 5$. (Do not use the result $\chi(G) \leq 4$.)