應數系博士班資格考－演算法

97年2月

1. Solve the recurrence $T(n) = 7T(n/3) + n^2$. (10%)

2. Solve the recurrence $T(n) = T(n^{1/2}) + 1$. (10%)

3. A sequence of n operations is performed on a data structure. The ith operation cost
   i is an exact power of 2, and 1 otherwise. Determine the amortized cost per operati
   (15%)

4. The 0-1 knapsack problem is described as follows: A thief robbing a store finds n ite
   the ith item is worth v_i dollars and weights w_i pounds, where v_i and w_i are intege
   He want to takes as valuable a load as possible, but he can carry at most W pou
   in his knapsack for some integer W. Which items should he take?
   Give a dynamic-programming solution to the 0-1 knapsack problem that runs in $O(n$
   time. (15%)

5. Arbitrage is the use of discrepancies in currency exchange rates to transform one u
   of a currency into more than one unit of the same currency. For example, suppo
   that 1 U.S. dollar buy 46.4 Indian rupees, 1 Indian rupee buys 2.5 Japanese yen, a
   1 Japanese yen buys 0.0091 U.S. dollars. Then, by converting currencies, a trader c
   start with 1 U.S. dollar and buy $46.4 \times 2.5 \times 0.0091 = 1.0556$ U.S. dollars, thus turni
   a profit of 5.56 percent. Suppose that we are given n currencies $c_1, c_2, \ldots, c_n$ and
   $n \times n$ table R of exchange rates, such that one unit of currency c_i buys $R[i, j]$ units
   currency c_j.
   Give an efficient algorithm to determine whether or not there exists a sequence
   currencies $\langle c_{i_1}, c_{i_2}, \ldots, c_{i_k} \rangle$ such that $R[i_1, i_2] \times R[i_2, i_3] \times \ldots \times R[i_{k-1}, i_k] \times R[i_k, i_1] >$
   Give the running time of your algorithm. (20%)

6. The subgraph-isomorphism problem takes two graphs G_1 and G_2 and asks whether
   is a subgraph of G_2. Show that the subgraph-isomorphism problem is NP-complet
   (15%)

7. Show that $\lceil 3n/2 \rceil - 2$ comparisons are necessary in the worst case to find both maximum and minimum of n numbers. (15%)