97學年度第一學期博士班資格考---圖　論

97.9.18

%In this test we follow the definitions and notation in West book.

1. (10%) Prove that If G is a graph, then $\chi'G) \le \frac{3}{2}\Delta(G)$.
2. (10%) Prove that If T is a tree with n vertices, then $\chi(T;k) = k(k-1)^{n-1}$.
3. (10%) Prove that If a graph G has degree sequence $d_1 \ge \cdots \ge d_n$, then $\chi(G) \le 1 + \max_i \min\{d_i, i-1\}$.
4. (10%) Prove that A graph G having at least three vertices is 2-connected if and only if for each pair $u,v \in V(G)$ there exist internally disjoint $u,v-$paths in G.
5. (10%) Prove that Every 3-regular graph with no cut-edge has a 1-factor.
6. (10%) Prove that For $k > 0$, every $k-$regular bipartite graph has a perfect matching.
7. (10%) Prove that A matching M in a graph G is a maximum matching in G if and only if G has no M - augmenting path.
8. (10%)Which of the following are graphic sequences? Provide a construction or a proof of impossibility for each. (a). (5, 5, 5, 3, 2, 2, 1, 1) (b). (5, 5, 4, 4, 2, 2, 1, 1) (a). (5, 5, 5, 4, 2, 1, 1, 1)
9. (10%)Prove that If G and H are two simple graphs with vertex set V, then $d_G(v) = d_H(v)$ for every $v \in V$ if and only if there is a sequence of 2-switches that transforms G into H.
10. (10%)Given positive integers $d_{1,\dots,}d_n$ summing to $2n-2$, there are exactly f(n) trees with vertex set $[n]$ such that vertex i has degree d_1, for each i. Find function f(n).