

應數系博士班資格考 - 演算法

97年2月

1. Solve the recurrence $T(n) = 7T(n/3) + n^2$. (10%)
2. Solve the recurrence $T(n) = T(n^{1/2}) + 1$. (10%)
3. A sequence of n operations is performed on a data structure. The i th operation cost i if i is an exact power of 2, and 1 otherwise. Determine the amortized cost per operation. (15%)
4. The 0-1 knapsack problem is described as follows: A thief robbing a store finds n items, the i th item is worth v_i dollars and weights w_i pounds, where v_i and w_i are integers. He want to takes as valuable a load as possible, but he can carry at most W pounds in his knapsack for some integer W . Which items should he take?
Give a dynamic-programming solution to the 0-1 knapsack problem that runs in $O(nW)$ time. (15%)
5. Arbitrage is the use of discrepancies in currency exchange rates to transform one unit of a currency into more than one unit of the same currency. For example, suppose that 1 U.S. dollar buy 46.4 Indian rupees, 1 Indian rupee buys 2.5 Japanese yen, and 1 Japanese yen buys 0.0091 U.S. dollars. Then, by converting currencies, a trader can start with 1 U.S. dollar and buy $46.4 \times 2.5 \times 0.0091 = 1.0556$ U.S. dollars, thus turning a profit of 5.56 percent. Suppose that we are given n currencies c_1, c_2, \dots, c_n and an $n \times n$ table R of exchange rates, such that one unit of currency c_i buys $R[i, j]$ units of currency c_j .
Give an efficient algorithm to determine whether or not there exists a sequence of currencies $\langle c_{i_1}, c_{i_2}, \dots, c_{i_k} \rangle$ such that $R[i_1, i_2] \times R[i_2, i_3] \times \dots \times R[i_{k-1}, i_k] \times R[i_k, i_1] > 1$.
Give the running time of your algorithm. (20%)
6. The subgraph-isomorphism problem takes two graphs G_1 and G_2 and asks whether G_1 is a subgraph of G_2 . Show that the subgraph-isomorphism problem is NP-complete. (15%)

7. Show that $\lceil 3n/2 \rceil - 2$ comparisons are necessary in the worst case to find both the maximum and minimum of n numbers. (15%)