93學年度博士班資格考－設計理論

> **Definition 1.** Let X be a finite set and $\mathscr{B}$ is a set of subsets (which may be of different sizes) of X. We say that $(X,\mathscr{B})$ is a pairwise balanced design (abbreviated as a PBD) if each pair of distinct elements of X occurs together in exactly one block in $\mathscr{B}$.
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> **Definition 2.** An affine plane is a PBD $(P,\mathscr{L})$ with extra conditions specified below, where elements of P are called points, sets in $\mathscr{L}$ are called lines, and two lines $l_1,l_2 \in \mathscr{L}$ with $l_1 \cap l_2 = \varnothing$ are said to be parallel. The conditions are:
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> (i) P contains at least one subset of 4 points with no 3 of which are contained in the same line;
>
> (ii) given a line l and a point $p \notin l$, there is exactly one line of $\mathscr{L}$ containing p which is parallel to l.

20% (1) Let $(P,\mathscr{L})$ be an affine plane. If $n \geq 2$, prove that the following statements are equivalent:

(a) There is a line in $\mathscr{L}$ containing n points.

(b) Every line contains n points.

(c) There are exactly $n+1$ lines.

12% (2) (a) Let $(X,\mathscr{B})$ be a 2-(v,k,λ) design. Assume that $|B_1 \cap B_2| \leq 2$ for any two distinct blocks $B_1, B_2 \in \mathscr{B}$. Prove that $k \leq (3+\sqrt{4v-7})/2$.

12% (b) Let D be a t-design and let s be a positive integer with $s < t$. Prove that D is also an s-design.

> We call a Latin square *self-orthogonal* if it is orthogonal to its own transpose.

12% (3) (a) Prove that there is no self-orthogonal Latin square of order 3.

12% (b) Find self-orthogonal Latin squares of orders 4 and 5.

12% (4) Let σ be an automorphism of a symmetric 2-(v,k,λ) design which fixes m points and n blocks. Show that $m = n$.

8% (5) (a) State Bruck-Ryser-Chowla theorem.

12% (b) Use Bruck-Ryser-Chowla theorem to show that there is no symmetric design with parameters $(29,8,2)$.