國立交通大學 應用數學系

博士班學生資格考試

〈演算法〉

2004, September

(9 problems in 3 pages)

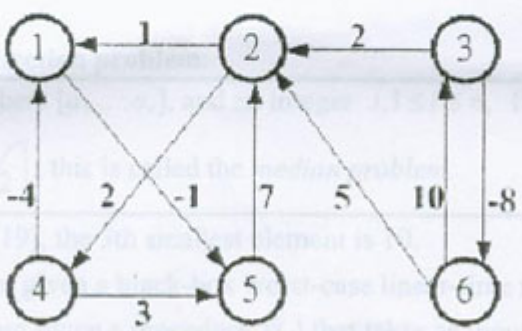
§ Express your algorithms at the proper level of detail. Give enough details to clearly present your solution, but not so many that the main ideas are obscured. English is often a good way to express an algorithm; pseudocode is good for communicating complex control structure. §

1. (10%) Give asymptotic **upper** and **lower** bounds for $T(n)$ in the following recurrence

$$T(n) = 2T(n/2) + n(\log n)^3$$

Make your bounds as tight as possible and justify your answers.

2. (10%) Run the **Floyd-Warshall algorithm** on the weighted, directed graph shown in the following figure. Show the matrix $D^{(k)}$ that results for each iteration of the outer loop.



3. (20%) The **subset sum problem** is: Given a set of n positive integers, $S=\{x_1,x_2,\ldots,x_n\}$ and a positive integer W determine whether there is a subset $S' \subseteq S$, such that the sum of the elements in S' is equal to W. Give a dynamic programming solution to the subset sum problem that runs in $O(nW)$ time. Justify the correctness and running time of your algorithm.

4. (10%) The **set partition problem** takes as input a set S of integers. The question is whether the integers can be partitioned into two sets A and $\bar{A} = S - A$ such that $\sum_{x \in A} x = \sum_{x \in \bar{A}} x$. Show that the set partition problem is NP-complete. You may use the fact that the subset sum problem is NP-complete.

(b) Assume you are given an array of n integers in the range $\{1,\ldots,(\log n)^{\log n}\}$. Show how to sort this array in time $O(n \log\log n)$.

8. (5%) Professor Nobody at NCTU recently announced his discovery: the Bogon heaps. Bogon heaps achieve the same amortized bounds as Fibonacci heaps, except for `delet_min`, which is implemented in $O(\log\log n)$ comparison. Do you believe in Professor Nobody's discovery? Why or Why not?

9. (5%) Show how to sort 5 elements, say Y_1, Y_2, Y_3, Y_4, Y_5, within 7 comparisons.