博士班資格考

Ph. D. Qualification Examination

Algorithms

P2 年 P月

1. (10%) Let P be a set of n points on the 2-dimensional Euclidean plane. Let T be a minimum spanning tree of P and let U be an optimal traveling-salesman tour of P.
   (a) Prove that $|T| \leq |U|$, where $|T|$ and $|U|$ are the lengths of T and U, respectively.
   (b) Prove that $|U| \leq 2|T|$.

2. (10%) Describe an algorithm that, given n integers in the range of 1 to k, preprocesses its input and then answers any query about how many of the n integers fall into range $[a \ldots b]$ in $O(1)$ time. Your preprocessing algorithm should perform in $O(n+k)$ time.

3. (10%) The knapsack problem is defined as follows: Given positive integers P_1, $P_2, \ldots, P_n$, $W_1, W_2, \ldots, W_n$, and M, find $X_1, X_2, \ldots, X_n$, $0 \leq X_i \leq 1$ such that

$$\Sigma P_i X_i \text{ is } \textit{maximized}$$

$$\textit{subject to } \Sigma W_i X_i \leq M.$$

   Given a greedy method to find an optimal solution of the above problem and prove its correctness.

4. (20%) Let $G=(V, E)$ be an undirected graph, in which each edge $e \in E$ has a weight $w(e)$. Suppose that G is represented by adjacency lists.
   (a) Design an efficient algorithm that constructs an arbitrary spanning tree of G. What's the time complexity of your algorithm?
   (b) Design an efficient algorithm that constructs a spanning tree of G whose largest edge weight is minimum over all spanning tree of G. What's the time complexity of your algorithm?

5. (10%) Give asymptotic upper and lower bounds for $T(n)$ in each of the following recurrences. Assume that $T(n)$ is constant for $n \leq 2$. Make your bounds as tight as possible. and justify your answer.

(a) $T(n)=3T(n/2)+n\lg n$.

(b) $T(n)=T(\sqrt{n})+1$.

6. (20%) Given two strings $x[1..m]$ and $y[1..n]$ and a given set of operation costs, the *edit distance* between x and y is the least expensive transformation sequence that converts x to y. Suppose the given set of operations and associated costs are listed as follows:

Operation	Description	Associated Costs
Delete	Delete a character	C_d
Replace	Replace a character	C_r
Copy	Copy a character	C_c
Insert	Insert a character	C_i
Twiddle	Interchange two adjacent characters	C_t
Kill	Kill to end of line	C_k

For example, the following table shows how to use the above operations to change the source string x="algorithm" into a string y="altruistic":

Operation	Target string	Source string
Copy a	a	lgorithm
Copy l	al	gorithm
Replace g by t	alt	orithm
Delete o	alt	rithm
Copy r	altr	ithm
Insert u	altru	ithm
Insert i	altrui	ithm
Insert s	altruis	ithm
Twiddle it to ti	altruisi	hm
Insert c	altruistic	hm
Kill hm	altruistic	

The cost of the above transformation sequence is $3C_c+C_r+C_d+4C_i+C_t+C_k$.

(a) Describe a dynamic-programming algorithm to find the *edit distance* from $x[1..m]$ to $y[1..n]$ using *Insert*, *Delete* and *Replace* operations

(b) Describe an algorithm like (a) but use all operations listed in above table.

7. (20%) For each of the following statement, determine whether it is correct or not.

(a) The lower bound of NP-hard problem is exponential if and on if P ≠NP is proved.

(b) The lower bound of NP-hard problem is exponential once if P ≠NP is proved.

(c) Suppose that it is proved that the lower bound of the satisfiability problem is polynomial, we can conclude that P=NP.

(d) It is proved that the problem of determining whether a given number is prime or not can be solved in polynomial time by a deterministic algorithm.

(e) It is proved that the problem of determining whether a given number is prime or not is NP-complete.