博士班資格考一圖論

92年2月

Notation: $\chi(G)$: the chromatic number of G.

$\chi'(G)$: the edge chromatic number of G.

$\chi(G; x)$: the chromatic polynomial function of x.

$T_{n,r}$ (Turan' graph) : the complete r-partite graph with n vertices that has b parts of size a+1 and r-b parts of size a, where a=[n/r] and b =n-ra.

$\Delta(G)$: the maximum degree of vertices of G.

L(G) : the line graph of G.

A: You are only required to choose any one from the following two problems.

1. Find two non-isomorphic graphs which have the same eigenvalues with the same multiplicities. (Biggs, Page 12)
2. Prove the following theorem (sachs, 1967) If G is a regular graph of degree k with n vertices and m = 1/2(nk) edges, then
   $\chi(L(G);x) = (x+2)^{m-n}\chi(G;x+2-k)$. (Biggs, Page 19)

B: You are only required to prove any three from the following seven theorems

1. Turan's theorem. ([1941]) Among the n-vertex simple graphs with no r+1-clique, $T_{n,r}$ has the maximum number of edges. (West, Page 34).
2. [Caylet Formula, 1889] : There are n^{n-2} trees with vertex set $\{1,2,\ldots,n\}$. (West, Page 63)
3. [P. Hall , 1935]) : If G is a bipartite graph with bipartition X,Y, then G has a matching of X into Y if and only if $|N(S)| \geq |S|$ for all $S \subseteq X$. (West, Page 100)
4. [Max-flow Min-Cut theorem , 1956]: In every network, the maximum value of a feasible flow equals the minimum capacity of a source/sink cut. (West, Page 162)
5. [Brook Theorem, 1941]. If G is a connected graph other than an odd cycle or a clique graph, then $\chi(G) \leqq \Delta(G)$. (West, Page 179).
6. Prove the following theorem (Vizing [1964]-Gupta[1966]) : If G is simple, then either $\chi' = \Delta$ or $\chi' = \Delta+1$.(West, Page 210).
7. [Grinberg , 1968] : If G is a loopless plane graph with a Hamiltonian cycle C, and G has f'_i faces of length i inside C and f''_i faces of length i outside C, then $\Sigma_i (i-2)(f'_i - f''_i) = 0$. (West, Page 275)