Q. Exam. Ordinary Differential Equations

SPRING, 2003

92年2月

1. (15 pts) Classify all possible flows for the equation

$$\frac{d}{dt}x = a_0 + a_1 x + a_2 x^2 + x^3,$$

on $\mathbf{R}$.

2. (18 pts) Rewrite the 2-dimensional system

$$\begin{cases} \frac{dx}{dt} = x - y - x(x^2+y^2) + \frac{xy}{\sqrt{x^2+y^2}}, \\ \frac{dy}{dt} = x + y - y(x^2+y^2) - \frac{x^2}{\sqrt{x^2+y^2}} \end{cases}$$

in polar coordinates and sketch the phase portrait in (x, y)-plane.

3. (22 pts) Find all the equilibria for the Hamiltonian system

$$\begin{cases} \frac{dq_1}{dt} = p_1, \\ \frac{dq_2}{dt} = p_2, \\ \frac{dp_1}{dt} = -q_1 - 2q_1q_2, \\ \frac{dp_2}{dt} = -q_2 - q_1^2 + q_2^2. \end{cases}$$

Do stability analysis for one of the stable equilibria and for one of the unstable equilibria.

4. (15 pts) Consider the system in $\mathbf{R}^2$

$$\frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = A \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix} e^t,$$

where A has real eigenvalues λ_1, λ_2 (both$\neq 1$) and $q_1(t)$, $q_2(t)$ are polynomials of degree ≤ 2. Prove that there exists a solution of the form

$$\begin{bmatrix} x \\ y \end{bmatrix}(t) = \begin{bmatrix} p_1(t) \\ p_2(t) \end{bmatrix} e^t,$$

where $p_i(t)$ are polynomials of degree ≤ 2.

5. (15 pts) Consider the equation in $\mathbf{R}$

$$\frac{d}{dt}x = (\delta + a(t))x,$$

where $a(t)$ is a periodic function with 2π. Find the (Floquet) characteristic exponent and discuss the stability of the origin $x = 0$.

6. (15 pts) Show that the phase portrait of

$$x'' - x'(1 - 3x^2 - 2(x')^2) + x = 0$$

has a limit cycle. $(' = \frac{d}{dt})$