博士班資格考 103年9月

NCTU Department of Applied Mathematics

Discrete Mathematics Qualifying Examination, September 2014

Problem 1.(15pts) Let χ_G be the chromatic polynomial of a finite graph G and let $\omega(G)$ be the number of acyclic orientations of G. Prove that $\omega(G) = (-1)^{|V(G)|}\chi_G(-1)$.

Problem 2.(15pts) Let G be a graph in which all vertices have degree ≤ 5 and such that K_6 is not a subgraph of G. Prove that $\chi(G) \leq 5$. You must prove any theorems used in your proof.

Problem 3.(15pts) Let G be a bipartite graph with vertex set $X \cup Y$, where every edge has one endpoint in X and one in Y. Suppose that $|\Gamma(A)| \geq |A|$ for every $A \subseteq X$. Prove that there is a complete matching from X to Y in G.

Problem 4.(15pts) Let $A_k = \{k-1, k, k+1\} \cap \{1, 2, \ldots, n\}$, $k = 1, 2, \ldots, n$. Let S_n denote the number of SDR's of the collection $\{A_1, A_2, \ldots, A_n\}$. Determine S_n and $\lim_{n\to\infty} S_n^{1/n}$.

Problem 5.(15pts) State and prove the Bruck-Ryser-Chowla theorem.

Problem 6.(15pts) State and prove the Ford-Fulkerson theorem (this theorem is also referred to as the maxflow-mincut theorem).

Problem 7.(10pts) Let $A_1, A_2, \ldots, A_m$ be subsets of $\{1, 2, \ldots, n\}$ such that A_i is not a subset of A_j if $i \neq j$. Prove that $m \leq \binom{n}{\lfloor n/2 \rfloor}$.