2013-14 Topic in Differential Geometry

It will cover methods of global analysis and fundamental results in differential geometry, including, differential equations on a manifold and the relation between curvature and topology of a Riemannian manifold. At the end, we will focus on problems concerning different areas of differential geometry. The first semester will cover :

1. First chapter : comparison theorems and gradient estimates for harmonic functions : Gradient estimates and the Harnack inequality are obtained under curvature assumption of lower bounds on Ricci curvatures. As its applications, we will give a general result on complete manifolds of non-negative Ricci curvature, including Cheeger-Gromoll splitting theorem.

2. Second chapter: harmonic functions on manifolds of negative curvature : We will discuss the existence of positive harmonic functions on a complete, simply connected Riemannian manifold which may or may not have negative sectional curvature everywhere. The mean value inequality for subharmonic functions and existence of a global Green's function are also discussed.

3. Third chapter: eigenvalue problems : We will discuss the basic properties of eigenvalues, the Sobolev and isoperimetric inequality. The upper and lower bounds estimates for the first positive eigenvalue as well as gaps between eigenvalues on a compact manifold are given.

4. Fourth chapter : heat kernel in Riemannian manifolds : It will cover gradient estimates and Harnack inequality for the heat kernel. With its applications, the upper and lower bounds for the heat kernel are obtained under assumption of the Ricci curvature.

Reference : R. Schoen and S.-T. Yau, Lectures on Differential Geometry, International Press, 1994.

Time: 13:30 - 14:45, Tuesday,Thursday, September 10, 2013 - January 10, 2014
Room: 102, Astronomy and Mathematics Building
Speaker: Shing-Tung Yau   (Harvard University & National Taiwan University)