TA Training - Definitions and Theorems

August, 2026

1. State the definition of $\lim_{x \to a} f(x) = L$. How about other types, for multi-variable functions?

2. State the Intermediate Value Theorem/Extreme Value Theorem.

3. State the definition of "$f : I \subseteq \mathbb{R} \to \mathbb{R}$ is differentiable at $a \in I$." Generalize it into $\mathbb{R}^n$.

4. State the Mean Value Theorem (for derivatives).

5. State the Chain Rule. Generalize it into $\mathbb{R}^n$.

6. State the Inverse Function Theorem. State the version in $\mathbb{R}^n$.

7. State the L'Höpital's Rule.

8. State the Fundamental Theorem of Calculus.

9. State the Second Derivative Test in $\mathbb{R}^2$. Generalize it into $\mathbb{R}^n$.

10. State the Lagrange Multiplier Theorem (particularly for $f(x, y, z)$ with 2 constraints).

11. Let f be a real-valued function that is smooth near a point $a \in \mathbb{R}$.

    (a) What is the Taylor polynomial of f at a?

    (b) What are the radius of convergence and the interval of convergence for a power series?

    (c) State the definition of "f is real-analytic at a."

12. Investigate Fubini's Theorem:

    Theorem 1. *Let D be a rectangle region be contained in $\mathbb{R}^2$*

    $$D = \{(x, y) \mid a \leq x \leq b,\ c \leq y \leq d\}$$

    and let $f : D \subseteq \mathbb{R}^2 \to \mathbb{R}$ be integrable. If $f^y(x) = f(x, y)$ is integrable on $[a, b]$ for each $y \in [c, d]$, and if $g(y) = \int_a^b f(x, y)dx$ is integrable on $[c, d]$. Then the Riemann integral of f over D equals to the iterated integral, i.e.,

    $$\iint_D f(x, y)dA = \int_c^d \left(\int_a^b f(x, y)dx \right) dy.$$

13. Investigate the Change of Variables for Multiple Integrals:

    Theorem 2. *Given an open set U in $\mathbb{R}^n$, let $\varphi : U \to \mathbb{R}^n$ be one-to-one and continuously differentiable on U. If the Jacobian of φ, $J\varphi(\mathbf{x}) \neq 0$ on U, A is a Jordan measurable set with $\overline{A} \subseteq U$, and f is bounded and integrable on $\varphi(A)$, then $f \circ \varphi$ is integrable on A and*

    $$\int_{\varphi(A)} f(\mathbf{y})d\mathbf{y} = \int_A f(\varphi(\mathbf{x}))\ |J\varphi(\mathbf{x})|\ d\mathbf{x}.$$