應用數學系 114 學年度暑期「助教訓練課程」說明

一、 碩一新生均需參加。

（系上規定碩班學生需參加且完成助教訓練課程方可擔任助教）。

二、 時間：8/20(三)~8/21(四) 上午 10:00~12:00 及下午 1:30~4:00

地點：基礎科學教學研究大樓 204 教室。

三、 課程內容等相關資料會在 6 月底應用數學系網站最新消息中公佈，同學可上網查看。

四、 最慢請在 8/5(二)填寫 Google 表單報名，並 mail 大學部成績單電子檔至 calculus@math.nctu.edu.tw，以方便我們建檔並統計人數(以應用數學系三個組別碩士班學生優先)。

五、 Google 表單報名網址： https://forms.gle/TccSmJeAyDR1kdmW6

2025 NYCU-Math TA training

(1) **<u>The definition of limits</u>** Let $L \in \mathbb{R}$. We write

$$\lim_{x \to a} f(x) = L$$

if, for any $\epsilon > 0$, there is $\delta > 0$ such that

$$|f(x) - L| < \epsilon, \quad \forall 0 < |x - a| < \delta.$$

(2) **<u>The intermediate value theorem</u>** If $f : [a, b] \to \mathbb{R}$ is continuous, then, for any L between $f(a)$ and $f(b)$, there is $c \in (a, b)$ such that $f(c) = L$.

(3) **<u>The extremum value theorem</u>** If $f : [a, b] \to \mathbb{R}$ is continuous, then f attains its maximum and minimum values. That is, there are $\alpha, \beta \in [a, b]$ such that

$$f(\alpha) \leq f(x) \leq f(\beta), \quad \forall x \in [a, b].$$

(4) **<u>The definition of differentiation</u>** Let $a \in \mathbb{R}$. A function f is differentiable at a if

$$f'(a) := \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \quad \text{exists},$$

or equivalently if there is (unique) $M \in \mathbb{R}$ such that

$$\lim_{x \to a} \frac{|f(x) - (f(a) + M(x - a))|}{|x - a|} = 0.$$

In particular, $M = f'(a)$.

(5) **<u>The mean value theorem for derivatives</u>** Let f be a function defined on $[a, b]$. If f is continuous on $[a, b]$ and differentiable on (a, b), then there is $c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

(6) **<u>The chain rule</u>** If f is differentiable at x and g is differentiable at $f(x)$, then $g \circ f$ is differentiable at x and $(g \circ f)'(x) = g'(f(x))f'(x)$.

(7) **<u>The inverse function theorem</u>** Suppose $f : (a, b) \to \mathbb{R}$ is one-to-one and differentiable on (a, b). If $f'(x) \neq 0$, then f^{-1} is differentiable at $f(x)$ and

$$(f^{-1})'(f(x)) = \frac{1}{f'(x)}.$$

(8) **The l'Hospital's rule** Let f, g be differentiable functions defined on (a, b). Assume that

$$\lim_{x\to a^+} f(x) = \lim_{x\to a^+} g(x) \in \{0, \pm\infty\}, \quad \lim_{x\to a^+} \frac{f'(x)}{g'(x)} \in \mathbb{R} \cup \{\pm\infty\}.$$

Then,

$$\lim_{x\to a^+} \frac{f(x)}{g(x)} = \lim_{x\to a^+} \frac{f'(x)}{g'(x)}.$$

(9) **The fundamental theorem of calculus** Let $f : [a, b] \to \mathbb{R}$ be continuous.
  (1) If $F(x) = \int_a^x f(t)dt$, then F is an antiderivative of f on (a, b) and continuous on $[a.b]$.
  (2) If $G : [a, b] \to \mathbb{R}$ is an antiderivative of f on (a, b) and continuous on $[a, b]$, then $\int_a^b f(t)dt = G(b) - G(a)$.

(10) **The mean value theorem for integrals** If $f : [a, b] \to \mathbb{R}$ is continuous, then there is $c \in (a, b)$ such that

$$\int_a^b f(x)dx = f(c)(b - a).$$

(11) **Lagrange multipliers** Let $f, g_1, ..., g_k : \mathbb{R}^n \to \mathbb{R}$ be continuously differentiable functions with $k \le n$ and $E = \{x \in \mathbb{R}^n | g_i(x) = 0, \forall 1 \le i \le k\}$. Suppose $\nabla g_1(x), ..., \nabla g_k(x)$ are linearly independent for all $x \in E$. If f, restricted to E, attains its maximum or minimum at P, then there are constants $c_1, ..., c_k$ such that

$$\nabla f(P) = c_1 \nabla g_1(P) + \cdots + c_k \nabla g_k(P).$$

(12) **Taylor series and analyticity** Let f be a function which is infinitely differentiable at a. The Taylor series of f centered at a refers to the following series
  (1) The Taylor series of f centered at a refers to the following series

$$T(x) := \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x - a)^n.$$

  (2) f is analytic at a if there is $\epsilon > 0$ such that

$$f(x) = T(x), \quad \forall x \in (a - \epsilon, a + \epsilon).$$

(13) **Fubini's Theorem** Let D is a rectangle region be contained in $\mathbb{R}^2$

$$D = \{(x, y) \mid a \le x \le b,\ c \le y \le d\}$$

and let $f : D \subseteq \mathbb{R}^2 \to \mathbb{R}$ is integrable. If $f_y(x) = f(x, y)$ is integrable on $[a, b]$ for each $y \in [c, d]$, and if $g(y) = \int_a^b f(x, y)dx$ is integrable on $[c, d]$. Then the Riemann integral of f over D equals to the iterated integral

$$\int\int_D f(x,y)dA = \int_c^d \left(\int_a^b f(x,y)dx\right)dy$$

(14) <u>**Change of Variables for multiple integrals**</u> Given U is a open set in $\mathbb{R}^n$, let $g : U \to \mathbb{R}^n$ is one to one and continuously differentiable on U. If the Jacobian of g, $Jg(\mathbf{x}) \neq 0$ on U, if A is a Jordan measurable and $\overline{A} \subseteq U$, if f is bounded and integrable on $g(A)$, then $f \circ g$ is integrable on A and

$$\int_{g(A)} f(\mathbf{y})d\mathbf{y} = \int_A f(g(\mathbf{x}))\ |Jg(\mathbf{x})|\ d\mathbf{x}$$