# 應用數學系 109 學年度暑假「助教訓練課程」說明

一、 要申請擔任助教工作之碩一新生均需參加(系上規定碩班學生需參加且完成助教訓練課程方可擔任助教)。

二、 時間：8/4(二)~8/6(四) 上午 9:30~12:00 及下午 1:30~3:30

地點：基礎科學教學研究大樓 204 教室。

三、 課程人數上限 40 人，最慢請在 7/20(一)E-mail 相關報名資料檔(含報名資料表與大學部成績單)至 calculus@math.nctu.edu.tw，以方便我們建檔並統計人數(以應用數學系三個組別碩士班學生優先)。

四、 課程大綱為次頁列出之 12 項核心能力，請參加者先行準備，課程進行期間會直接考核並作為分配擔任助教的參考依據。

備註：相關事項將於應用數學系網站最新消息中公佈，同學可上網查看。

# 2020 NCTU-Math TA training

(1) **The definition of limits** Let $L \in \mathbb{R}$. We write

$$\lim_{x \to a} f(x) = L$$

if, for any $\epsilon > 0$, there is $\delta > 0$ such that

$$|f(x) - L| < \epsilon, \quad \forall 0 < |x - a| < \delta.$$

Question: How to define limits if $a = \infty$ and what does it mean if $L = \infty$?

(2) **The intermediate value theorem** If $f : [a, b] \to \mathbb{R}$ is continuous, then, for any $L$ between $f(a)$ and $f(b)$, there is $c \in (a, b)$ such that $f(c) = L$.

Question: Whether the continuity of $f$ can be removed or $[a, b]$ can be replaced by any of its subsets that contain $a$ and $b$?

(3) **The extremum value theorem** If $f : [a, b] \to \mathbb{R}$ is continuous, then $f$ attains its maximum and minimum values. That is, there are $\alpha, \beta \in [a, b]$ such that

$$f(\alpha) \le f(x) \le f(\beta), \quad \forall x \in [a, b].$$

Question: What's the extremum value theorem for functions of multiple variables?

(4) **The definition of differentiation** Let $a \in \mathbb{R}$. A function $f$ is differentiable at $a$ if

$$f'(a) := \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \quad \text{exists},$$

or equivalently if there is (unique) $M \in \mathbb{R}$ such that

$$\lim_{x \to a} \frac{|f(x) - [f(a) + M(x - a)]|}{|x - a|} = 0.$$

In particular, $M = f'(a)$.

Question: How to define the differentiation of multiple-variable functions?

(5) **The mean value theorem for derivatives)** Let $f$ be a function defined on $[a, b]$. If $f$ is continuous on $[a, b]$ and differentiable on $(a, b)$, then there is $c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Question: Can one remove the continuity or differentiability of $f$? Does the mean value theorem apply for vector functions?

(6) **The chain rule** If $f$ is differentiable at $x$ and $g$ is differentiable at $f(x)$, then $g \circ f$ is differentiable at $x$ and $(g \circ f)'(x) = g'(f(x))f'(x)$.

Question: What is the chain rule for $g \circ f$ if $x \in \mathbb{R}^m$, $f(x) \in \mathbb{R}^n$ and $g(f(x)) \in \mathbb{R}^p$ with $m, n, p \in \mathbb{N}$?

(7) **<u>The inverse function theorem</u>** Suppose $f : (a,b) \to \mathbb{R}$ is one-to-one and differentiable on $(a,b)$. If $f'(x) \neq 0$, then $f^{-1}$ is differentiable at $f(x)$ and

$$(f^{-1})'(f(x)) = \frac{1}{f'(x)}.$$

Question: What is the inverse function theorem for $f : U \to \mathbb{R}^n$, where $U$ is an open subset of $\mathbb{R}^n$?

(8) **<u>The l'Hospital's rule</u>** Let $f, g$ be differentiable functions defined on $(a,b)$. Assume that

$$\lim_{x\to a^+} f(x) = \lim_{x\to a^+} g(x) \in \{0, \pm\infty\}, \quad \lim_{x\to a^+} \frac{f'(x)}{g'(x)} \in \mathbb{R} \cup \{\pm\infty\}.$$

Then,

$$\lim_{x\to a^+} \frac{f(x)}{g(x)} = \lim_{x\to a^+} \frac{f'(x)}{g'(x)}.$$

Question: Does the l'Hospital's rule hold for the case that $a \in \{\pm\infty\}$?

(9) **<u>The fundamental theorem of calculus</u>** Let $f : [a,b] \to \mathbb{R}$ be continuous.

(1) If $F(x) = \int_a^x f(t)dt$, then $F$ is an antiderivative of $f$ on $(a,b)$ and continuous on $[a.b]$.

(2) If $G : [a,b] \to \mathbb{R}$ is an antiderivative of $f$ on $(a,b)$ and continuous on $[a,b]$, then $\int_a^b f(t)dt = G(b) - G(a)$.

Question: Can one remove the continuity of $f$?

(10) **<u>The mean value theorem for integrals</u>** If $f : [a,b] \to \mathbb{R}$ is continuous, then there is $c \in (a,b)$ such that

$$\int_a^b f(x)dx = f(c)(b-a).$$

Question: Can one remove the continuity of $f$?

(11) **<u>Lagrange multipliers</u>** Let $f, g_1, ..., g_k : \mathbb{R}^n \to \mathbb{R}$ be continuously differentiable functions with $k \leq n$ and $E = \{x \in \mathbb{R}^n | g_i(x) = 0, \forall 1 \leq i \leq k\}$. Suppose $\nabla g_1(x), ..., \nabla g_k(x)$ are linearly independent for all $x \in E$. If $f$, restricted to $E$, attains its maximum or minimum at $P$, then there are constants $c_1, ..., c_k$ such that

$$\nabla f(P) = c_1 \nabla g_1(P) + \cdots + c_k \nabla g_k(P).$$

Question: Can one remove the linear independency of $\nabla g_1, ..., \nabla g_k$?

(12) **<u>Taylor series and analyticity</u>** Let $f$ be a function which is infinitely differentiable at $a$. The Taylor series of $f$ centered at $a$ refers to the following series

(1) The Taylor series of $f$ centered at $a$ refers to the following series

$$T(x) := \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n.$$

(2) $f$ is analytic at $a$ if there is $\epsilon > 0$ such that

$$f(x) = T(x), \quad \forall a - \epsilon < x < a + \epsilon.$$

Question: What are the radius of convergence and interval of convergence of $T$? What is the Taylor series of function $f : \mathbb{R}^n \to \mathbb{R}$ at $a \in \mathbb{R}^n$?

# 報名資料表

姓名：

手機：

E-mail：

本人郵局局號帳號(或玉山銀行)：

(請附上大學成績單正本掃描檔或 PDF 檔)

請填寫報名資料並附上成績單(電子檔)後 E-mail 至

calculus@math.nctu.edu.tw