

# Numerical Solution of The Large-Scale Riccati Differential Equation

## Abstract:

Consider the large-scale Riccati differential equation (RDE):  
$$\dot{X} = A^\top X + XA - XGX + H, \text{ with } X(t_f) = X_f,$$
 where  $A \in \mathbb{R}^{n \times n}$  is large and structured,  $G = BR^{-1}B^\top$  and  $H = C^\top T^{-1}C$  are low-ranked and  $X_f$  is numerically low-ranked. We shall show that  $X(t)$  can be expressed in terms of the steady-state solution  $X_\infty$ , which is numerically low-ranked. The procedure leads to an efficient numerical method for the solution of the RDE. In addition,  $X(t)$  is also numerically low-ranked and the RDE can then be projected onto a much smaller Krylov subspace, leading to a novel Krylov subspace projection method for the RDE. Finally, we prove the inheritance of solvability of the projection method for the RDE, that the Lipschitz continuity of the original RDE implies that of the projected RDE.