The cutoff phenomenon for ergodic Markov processes

Abstract

The idea of cutoff phenomenon was introduced by D. Aldous and P. Diaconis to capture the fact that some ergodic Markov chains converge abruptly to their invariant distributions. The first example where a cutoff of total variation was proved is the random transposition Markov chain on the symmetric group studied by Diaconis and Shahshahani in 1981. One of the most precise and interesting cutoff result concerns repeated riffle shuffles which was proved by D. Aldous in 1983 and improved by D. Bayer and P. Diaconis in 1992.

In this talk, we consider the cutoff phenomenon in the context of families of ergodic Markov transition functions. This includes classical examples such as families of ergodic finite Markov chains and Brownian motions on families of compact Riemannian manifolds. We give criteria for the existence of a cutoff when convergence is measured in L^p-norm, $1 < p < \infty$. This allows us to prove the existence of a cutoff in cases where the cutoff time is not explicitly known. In the reversible case, for $1 < p \leq \infty$, we show that a necessary and sufficient condition for the existence of a max-L^p cutoff is that the product of the spectral gap by the max-L^p mixing time tends to infinity. Concerning the cutoff time, we give a formula on the L^2 mixing time for reversible cases using the spectral decomposition.