Lit-only σ-game and its dual game on a graph

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黃皜文

Graph puzzle

Wilson, Richard M., Graph puzzles, homotopy, and the alternating group. J. Combinatorial Theory Ser. B 16 (1974), 86-96.

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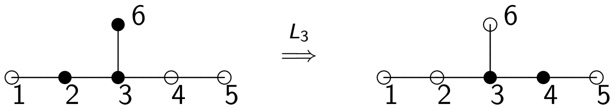
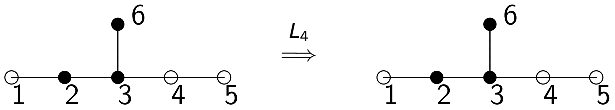
Lit-only σ-game on a graph





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矩陣建模

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2. Let A be the adjacency matrix of G. Then $L_i \in \mathrm{Mat}_{n\times n}(F_2)$ such that

$$L_i u = u + < e_i, u > Ae_i = u + e_i^T u A e_i = (I + Ae_i e_i^T) u.$$

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3. Hence $L_i = I + Ae_ie_i^T$ satisfies $L_i^2 = I$; in particular, L_i is invertible.
4. What is the matrix subgroup $\mathbf{W}(G)$ of $GL_n(F_2)$ that is generated by the matrices $L_1, L_2, \ldots, L_n$?

Proposition

If i is adjacent to j then $(L_iL_j)^3 = I$.

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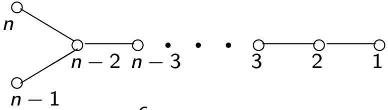
Theorem

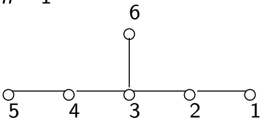
(Hau-wen Huang, W) If G is the Dynkin diagram of types A_n, D_n, E_6, E_7, or E_8 then $\mathbf{W}(G)$ is isomorphic to $W(G)/Z(W(G))$, where $W(G)$ is the Coxeter group associated with G, and $Z(W(G))$ is the center of $Z(W(G))$.

$A_n (n \geq 1)$

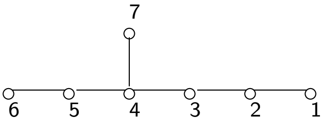
$D_n (n \geq 4)$



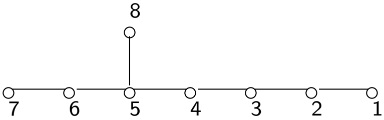
E_6



E_7



E_8



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(Yaokun Wu (吳耀琨)) If G is a tree with n vertices and $L(G)$ is the line graph of G, then $\mathbf{W}(L(G))$ is isomorphic to the symmetric group S_n.

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Theorem

(Hau-wen Huang, W) If G is a connected graph with $n \geq 3$ vertices and m edges and $L(G)$ is the line graph of G, then

$$\mathbf{W}(L(G)) \cong \begin{cases} (\mathbb{Z}/2\mathbb{Z})^{(n-1)(m-n+1)} \rtimes S_n, & \text{if } n \text{ is odd;} \\ (\mathbb{Z}/2\mathbb{Z})^{(n-2)(m-n+1)} \rtimes S_n, & \text{if } n \text{ is even,} \end{cases}$$

where $\mathbb{Z}$ is the additive group of integers.

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3. The **maximum-orbit-weight** $M(G)$ of a graph G is the maximum number of orbit weights $w(O)$ among all orbits $O \subseteq F_2^n$.
4. $M(G)$ is also called the **minimum light number** of G.

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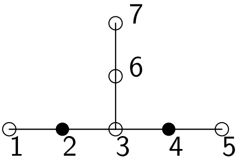
Theorem

(Hsin-Jung Wu (吳欣融)), (Xinmao Wang (王新茂)and Yaokun Wu) If G is a tree then $M(G) \leq \lceil \ell/2 \rceil$, where ℓ is the number of leafs in G.

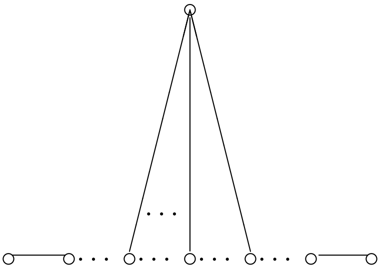


$$M(G) = 2.$$

Theorem

(Hau-wen Huang) If G is a tree with a perfect matching then $M(G) = 1$.

The graph G containing induced path P_{n-1}



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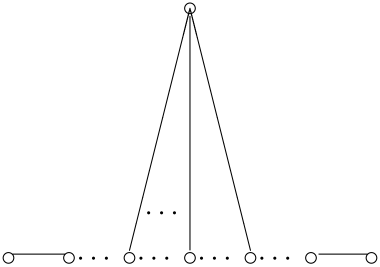

To generalize the results in simply-laced Dynkin diagrams, the orbits description of the above G is completed, and those graphs G with $M(G) = 1$ are characterized.

Lit-only σ-game orbits 描述及最少黑點數 $M(G)$ 的決定不太容易, 即使完成也不一定能看出它們與圖結構的關係. 有時候假設 G 上有一些 loops, 較有組合意義的結果.

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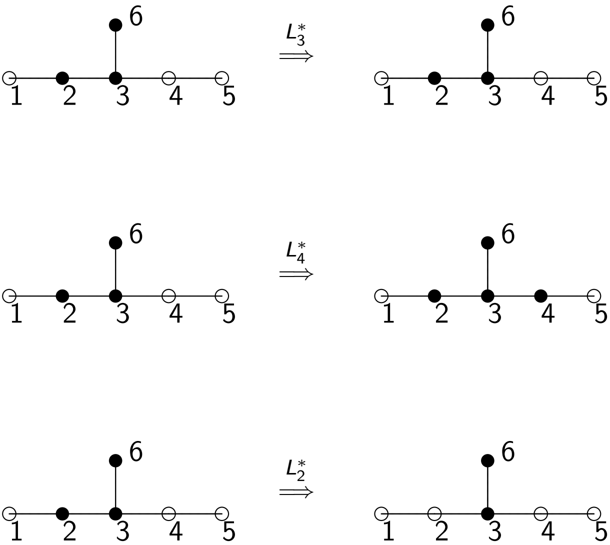
Reeder's game on a graph



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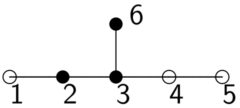
我們用 * 來表示 Reeder's game 這邊相對應的東西, 如 L_i^*.

不變量

For $u \in F_2^n$,

$$INV^*(u) := |\{i \mid u_i = 1\}| + |\{e = ij \mid u_i = u_j = 1\}| \pmod 2.$$

u :



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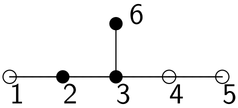

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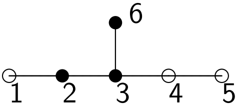

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Note that

$$INV^*(u) = INV^*(L_i^* u).$$

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Theorem

(Mark Reeder, 2005) Suppose G is a tree with odd number of matchings of size $\lfloor n/2 \rfloor$, but not a path. If O^* is a Reeder's game orbit, then exactly one of the the following (i)-(iii) holds.

(i) $O^* = \{u\}$ for some $u \in F_2^n$ with $Au = 0$; (只有單一不動點)

(ii) $O^* = \{u \in F_2^n \mid INV^*(u) = 1\}$. (黑點構成奇數個 trees, $w(O^*) = 1$)

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這定理的結論, 等價於一狀態如有三個以上的黑點, 且在 Reeder's game 中能動, 則黑點數一定能降到 1 或 2

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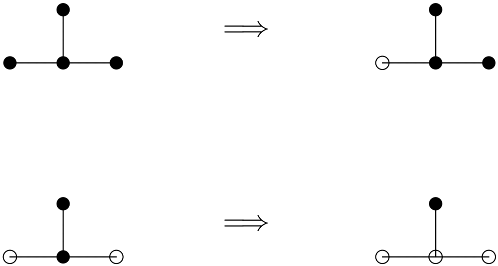
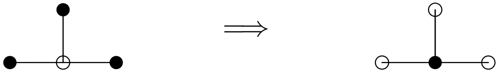
此定理可以對所有不是 path 的 tree 都成立嗎?

林育生

黑葉漂白



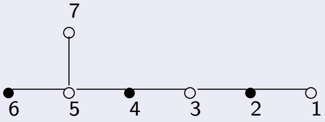
分支瘦身



一夫當關

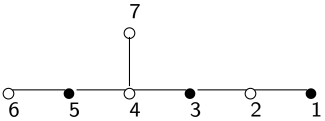
Exercise

(Hau-wen Huang)



永遠無法降為 1 個黑點

以退爲進



可降爲 1 個黑點

Binary star (雙星) $P_{t,a,b}$, $t \geq 4$

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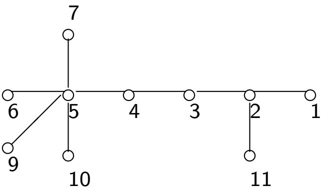
$P_{6,3,1}$



$P_{n,0,0} = P_n$ is a path of length $n - 1$.

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真的可以嗎?

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眞的可以嗎? 怎麼樣都說不清楚.

Stars, $P_{4,a,b}$, 及 $P_{5,a,b}$ 的直徑都不太長, 塞了三個以上獨立的黑點, 如果還能動, 很容易去降黑點數.

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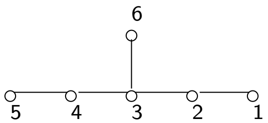
E_6



只要會證以下引理即可:

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Lemma

If G is a tree containing a subgraph isomorphic to E_6 , and O^* is a Reeder's game orbit, then exactly one of the the following (i)-(iii) holds.

(i) $O^* = \{u\}$ *for some* $u \in F_2^n$ *with* $Au = 0$*; (只有單一不動點)*

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(iii) $O^* = \{u \in F_2^n \mid Au \neq 0, INV^*(u) = 0\}$. *(黑點構成偶數個 trees,* $w(O^*) = 2$*)*

只要會證以下引理即可:

Lemma

If G is a tree containing a subgraph isomorphic to E_6 , and O^* is a Reeder's game orbit, then exactly one of the the following (i)-(iii) holds.

(i) $O^* = \{u\}$ for some $u \in F_2^n$ with $Au = 0$; (只有單一不動點)

(ii) $O^* = \{u \in F_2^n \mid INV^*(u) = 1\}$. (黑點構成奇數個 trees, $w(O^*) = 1$)

(iii) $O^* = \{u \in F_2^n \mid Au \neq 0, INV^*(u) = 0\}$. (黑點構成偶數個 trees, $w(O^*) = 2$)

Proof.

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5. 先假設 x 是 E_6 以外唯一的 leaf. 則 $G = P_{n-6} + E_6 + e$, 其中邊 e 連接 P_{n-6} 的端點到 E_6 某點.

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8. 所以必有另一個 leaf y 不在 E_6 上.

□

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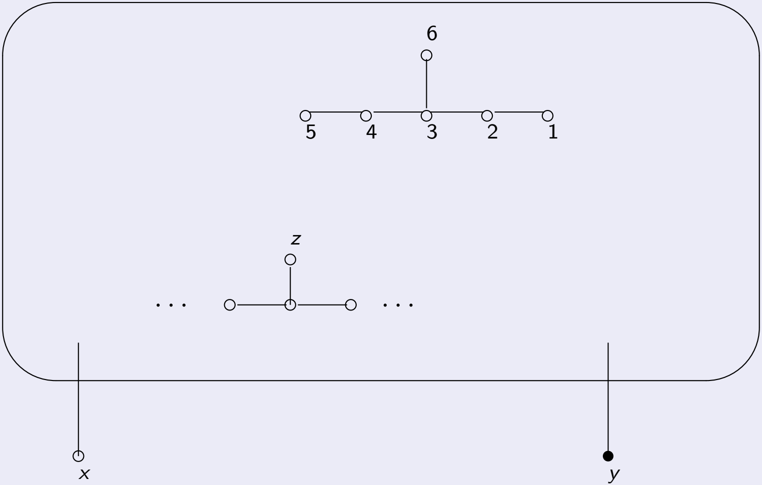

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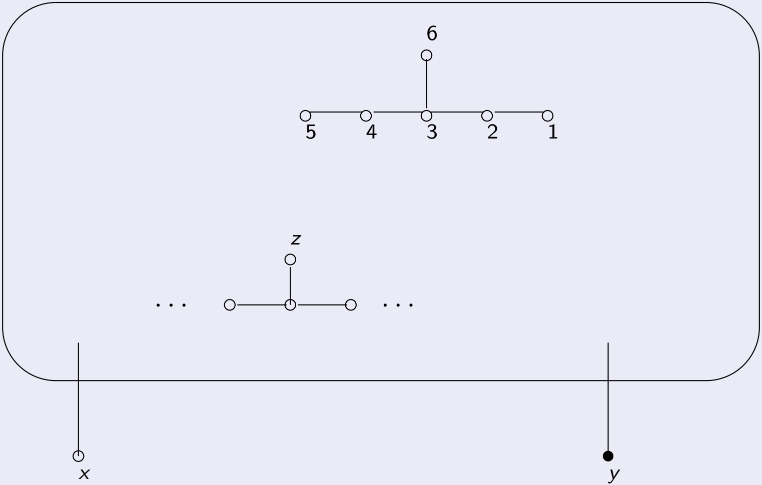

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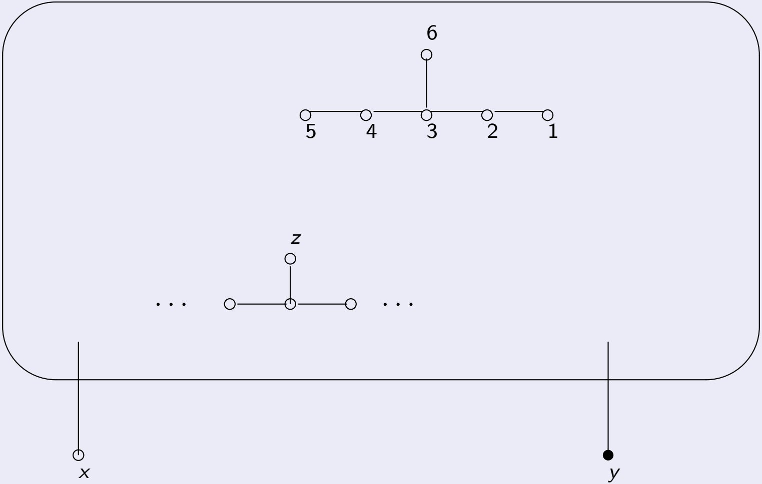

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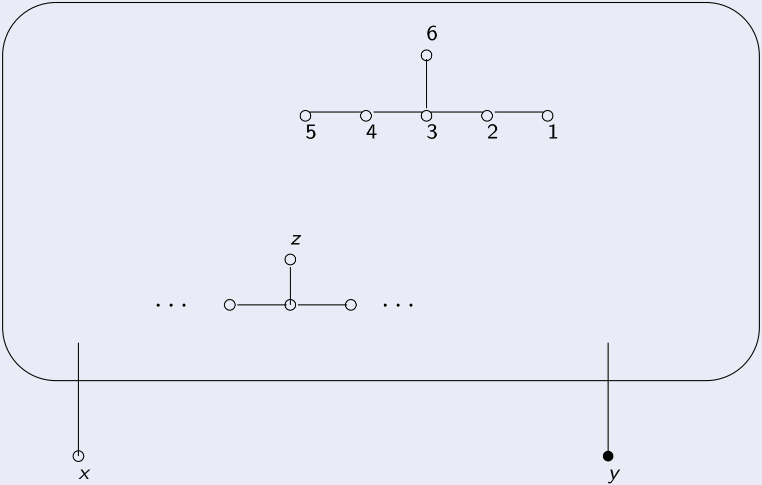

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但路徑中分支點的不在路徑上第三鄰居 z 必能動, 而可套歸納法於 $G - y$ 上.

這個簡單的證明似乎與我們得到結果的想法 "黑葉漂白", "分支瘦身", "一夫當關", "以退爲進" 完全無關,

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因爲痲煩的起始條件 E_6 及 $E_6 + P_{n-6} + e$ 的驗證, 我們引用了 Reeder 的結果, 其中也需要 $E_6 + P_{n-6} + e$ 不會是 path 這性質.

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E_6 是最小有 perfect matching 又不是 path 的 tree.

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$$L_i^* u = u + < Ae_i, u > e_i = u + e_i^T A u e_i = (I + e_i e_i^T A)u.$$

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每個人都能改變自己, 也就能改變別人.

單一黑點應該落在哪一 lit-only σ-軌道的組合描述

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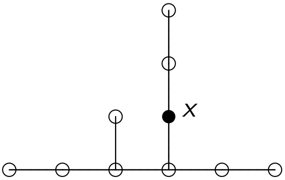

$\overset{A}{\Longleftarrow}$

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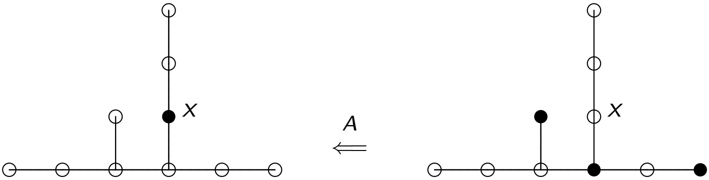

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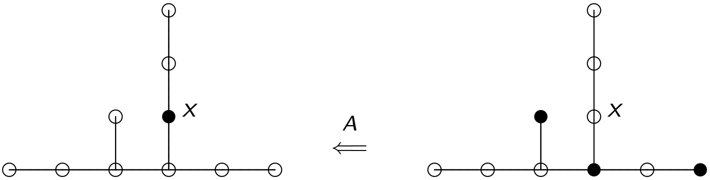

Let x, y be two vertices of a tree G with a perfect matching. The characteristic vectors $\widetilde{x}, \widetilde{y}$ of x, y respectively are in the same lit-only σ-orbit iff the number of matchings of size $\frac{n-2}{2}$ in $G - x$ and the number of matchings of size $\frac{n-2}{2}$ in $G - y$ have the same parity.

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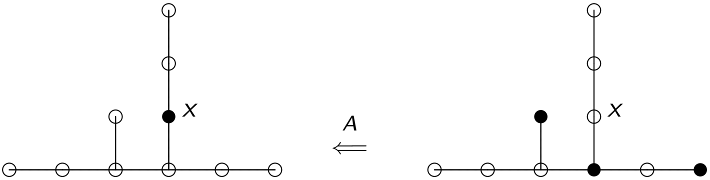

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單獨黑點落在哪一個軌道, 由白點中組成 maximum mathchings 的方法數之奇偶性決定.

Problem

給兩個以上黑點的 configuration 應該落在哪一 lit-only σ-game orbit 的組合描述.

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這個問題解答對 lit-only σ-game 上的不變量探討有幫助.

To be continued

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Thank you for your attention.