ICPAM-VAN 2018

11-13 SEPTEMBER 2018,
VAN YÜZÜNCÜ YIL UNIVERSITY
VAN-TURKEY

BOOK OF ABSTRACTS AND PROGRAMME

2ND INTERNATIONAL CONFERENCE ON PURE AND APPLIED MATHEMATICS

icpam.yyu.edu.tr

icpam2018@yyu.edu.tr
icpam2018.van@gmail.com

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Edited by Zeynep Kayar

Designed by Fatih Kutlu

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2nd International Conference on Pure and Applied Mathematics (ICPAM-VAN 2018)

BOOK OF ABSTRACTS

Van, TURKEY
September 11 - 13, 2018

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ALGEBRA

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Welcome

Dear Participants,

Welcome to 2nd International Conference on Pure and Applied Mathematics, ICPAM - VAN 2018, Van, Turkey. The conference is organized and is to be held at Van Yüzüncü Yıl University from September 11th to September 13th. This is the second ICPAM meeting in Van and the first one was organized in 2015. The reason for our 3-year long break is to be a host of another high involvement conference. We are happy to see you here in Van.

The main aim of this successive conference is to provide participants with an opportunity to exchange the latest information and ideas, and to encourage debate on many issues in international mathematical researches. During the conference you will certainly meet old and new colleagues, exchange ideas, develop new projects. You will also feel and enjoy the special atmosphere of Van and Turkey.

With 9 parallel sessions (totally 36 sessions) and 131 presentations and more than 150 participants from 11 countries, Algeria, China, Egypt, India, Iran, Pakistan, Portugal, Taiwan, Turkey, Ukraine, United Kingdom, as well as people from 37 different university from Turkey, ICPAM - VAN 2018 will provide a stimulating opportunity for a global interchange of ideas on recent advances in mathematics.

I would like to express my deep gratitude to Prof. Dr. Peyami BATTAL, President of Van Yüzüncü Yıl University, for his encouragement and support in all stages of this conference.

I am grateful to all the participants in the International Conference on Pure and Applied Mathematics, particularly the members of the Scientific and Organizing Committees, the referees and the authors for producing such a high standard conference.

The conference is almost entirely from the registration support of participants. Behind this, we are grateful to Rectorate of YYU, Faculty of Sciences of YYU and Abdullah Gül University for their financial support. We would also like to thank to Governor of Van for his generous support. Have a pleasant stay in Van.

Professor Cemil Tunç
Chair Organizer of ICPAM - VAN Organizing Committee

Acknowledgements

Dear Participants,

It is a great pleasure having this opportunity to thank all the people who made 2nd International Conference on Pure and Applied Mathematics (ICPAM-VAN 2018) possible.

We would like to to express our gratitude and deep appreciation to Prof. Dr. Cemil Tunc for his precious guidance, continuous encouragement, boundless energy which never seems to end and persuasive support throughout the organization. His great enthusiasm and belief in us make this conference possible.

We extend our sincere thanks to our colleagues who worked with us during the preparation of this conference for their help and support anytime we needed. We are grateful to all the members of Van Yüzüncü Yıl University Mathematics family, academic and administrative, who kindly help us at every stage of this project.

We would also like to thank to the sponsors, Van Yüzüncü Yıl University and Abdullah Gül University for their generous support.

The last but not the least, we would like to give our heartful thanks to participants for their understanding, patience and contributions which make our conference more successful. Since the conference is almost entirely from the registration support of participants, we are grateful for their financial support as well. Thank you all again for the unbelievable amount of support and understanding you have shared during this period. It means more than words can express. We were honored and happy to welcome you to in Van. We hope to see you again in Van, the city of the sun.

Zeynep Kayar (Van Yüzüncü Yıl University)
Ali Hakan Tor (Abdullah Gül University)

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16:50-17:10 **Vedat DÖRMA**, Bülent KARAKAŞ, Şenay BAYDAŞ
Kinematics of 4R and 2RPR mechanisms in Clifford algebra

Room 106 Chair: Cesim TEMEL

14:15-14:35 Elena BESPALOVA, **Nataliia YAREMCHENKO**
Quasilinearization method in problems of the subcritical deformation of flexible shell systems

14:35-14:55 **Ramazan OZARSLAN**, Erdal BAS, Ahu ERCAN
Singular eigenvalue problems via Hilfer derivative

14:55-15:15 Mehmet Giyas SAKAR, **Onur SALDIR**, Fevzi ERDOGAN
A numerical approach for time-fractional Kawahara equation with reproducing kernel method

15:15-15:35 Zeliha KÖRPINAR, **Fatih COŞKUN**
On the cubic nonlinear Shrodinger's equation with repulsive delta potential

15:35-15:50 Coffee Break

15:50-16:10 POSTER PRESENTATIONS

Room 106 Chair: Hayri TOPAL

16:10-16:30 **Sumeyra UCAR**, Nihal YILMAZ ÖZGÜR
Canonical finite Blaschke products and decomposibility

16:30-16:50 Roselaine Neves MACHADO, **Luiz Guerreiro LOPES**
Ehrlich-Aberth's type method with King's correction for the simultaneous approximation of polynomial zeros

16:50-17:10 **Omer KUSMUS**, I. Hakkı DENIZLER, Necat GORENTAS
Idempotent unit group in commutative group rings of direct products

Room 107 Chair: Mehmet Giyas SAKAR

14:15-14:35 Hıfsı ALTINOK, Mithat KASAP, **Derya DENİZ**
Strongly Cesaro summability of order β with respect to a modulus function

14:35-14:55 Hıfsı ALTINOK, **Derya DENİZ**, Mithat KASAP
Some properties of sequence class $S^{\beta}(\Delta, F, f)$ defined by a modulus function

14:55-15:15 **Mahmut KARAKUŞ**, Tunay BİLGİN
$\Lambda-$ matrix as a summability operator and completeness of certain normed spaces via weakly unconditionally Cauchy series

15:15-15:35 **Yasin KAYA**
The maximal function in Sobolev spaces

15:35-15:50 Coffee Break

15:50-16:10 POSTER PRESENTATIONS

September 12, 2018

Room 107 Chair: Mahmut KARAKUŞ

16:10-16:30 **Kazem HAGHNEJAD AZAR**
Some notes on the order-to-topology continuous operators

16:30-16:50 **Mehmet Giyas SAKAR**, Onur SALDIR, Fevzi ERDOGAN
Reproducing kernel method with Bernstein polynomials for fractional boundary value problems

16:50-17:10 **İsmail Hakkı DENİZLER**
An analogue of the Artin-Rees Lemma for Artinian modules

15:50-16:10 POSTER PRESENTATIONS

Mohammed Salah ABDELOUAHAB, Safa BOURAFA
On Caputo and Riemann-Liouville fractional-order derivatives with fixed memory length

Ecem ACAR, Aydın İZGİ
On approximation by generalized Bernstein-Durrmeyer operators

Filiz KANBAY, Nurten VARDAR
A fuzzy methodology on surface representation of greenhouse gas estimation

Mohammad Bagher MOGHIMI
Asymptotic aspect of some functional equations

Ilkay YASLAN KARACA, **Aycan SİNANOĞLU ARISOY**
Existence of positive solutions for second order impulsive boundary value problems on the half-line

	September 12, 2018 Wednesday								
Time	**Amphi 1**	**Amphi 2**	**Room 101**	**Room 102**	**Room 103**	**Room 104**	**Room 105**	**Room 106**	**Room 107**
Chair	**Y. YAYLI**								
9:30-10:15	C. W. WENG	Invited Speaker							
10:15-11:00	M. LAKESTANI	Invited Speaker							
11:00-11:15	Coffee Break								
Chair	**M. LAKESTANI**								
11:15-12:00	S. SHAHMORAD	Invited Speaker							
12:00-12:45	Y. YAYLI	Invited Speaker							
12:45-14:15	Lunch Break								
Chair		**N. GÖRENTAŞ**	**B. KALKAN**	**S. ERDUR**	**M. LUZUM**	**Ö. KÜSMÜŞ**	**İ. H. DENİZLER**	**C. TEMEL**	**M. G. SAKAR**
14:15-14:35		F. YÖRÜK DEREN	R. N. ÜREGEN	A. KHOJALI	H. KUŞAK SAMANCI	E. ERTEM AKBAŞ	U. DURAN	N. YAREMCHENKO	D. DENİZ
14:35-14:55		T. ŞENLİK ÇERDİK	J. AZAMI	H. GÜNAY AKDEMİR	E. ERKAN	A. KAZEMI PILEDARAQ	E. DEMİRCİ AKARSU	R. OZARSLAN	D. DENİZ
14:55-15:15		F. A. ÇETİNKAYA	R. VARMAZYAR	F. KUTLU	S. BAŞ	N. A. LONE	Ü. SARP	O. SALDIR	M. KARAKUŞ
15:15-15:35		A. AHMADKHANLU	S. İĞRET ARAZ	F. TUĞRUL	M. B. KAZEMI	N. ALSOY-AKGÜN	M. BEKAR	F. COŞKUN	Y. KAYA
15:35-15:50	Coffee Break								
15:50-16:10	Poster Presentations								
Chair		**F. YÖRÜK DEREN**	**R. YAZGAN**	**H. TAŞKESEN**	**F. KUTLU**	**C. W. WENG**	**Ş. BAYDAŞ**	**H. TOPAL**	**M. KARAKUŞ**
16:10-16:30		H. IŞIK	T. AKTÜRK	A. YILMAZ	G. SOYDAN	M. Ş. ALDEMİR	F. TUTAR	S. UCAR	K. HAGHNEJAD AZAR
16:30-16:50		E. ŞEN	T. AKTÜRK	N. ERDOGAN	S. ŞENYURT	M. Ş. ALDEMİR	B. KALKAN	L. G. LOPES	M. G. SAKAR
16:50-17:10		C. TEMEL	Z. KÖRPINAR	D. ARSLAN	T. YILDIRIM	S. EDİZ	V. DÖRMA	Ö. KÜSMÜŞ	İ. H. DENİZLER

Social Programme

September 13, 2018

8:00-16:00 SOCIAL PROGRAM

8:00 Departure from university campus
9:00-:10:30 Van Breakfast
10:30 Departure for Akhdamar Island
11:30-13:00 Visit to Akdamar Island
13:00 Departure for lunch
14:30-16:00 Lunch (in city centre)
16:00 Return to university campus

	September 13, 2018 Thursday
Time	Program
8:00	Departure from university campus
09:00-10:30	Van Breakfast
10:30	Departure for Akhdamar Island
11:30-13:00	Visit to Akhdamar Island
13:00	Departure for lunch
14:30-16:00	Lunch (in city centre)
16:00	Return to university campus

Abstracts of invited speakers

Nonlocality and fractional calculus: Finding the best kernel

DUMITRU BALEANU[1,2]

[1]Cankaya University, Ankara, Turkey
[2]Institute of Space Sciences, Bucharest, Romania

emails: [1]dumitru@cankaya.edu.tr

Fractional calculus deals with the study of so-called fractional order integral and derivative operators over real or complex domains, and their applications. During the last decades, the fractional differentiation has drawn increasing attention in the study of so-called anomalous behaviors, where scaling power law of fractional order appears universal as an empirical description of such complex phenomena [1, 3]. In my talk, for a given real world models, I will present and compare the importance of singular and nonsingular kernels in capturing the non-local effects. A special attention will be devoted to the Mittag-Leffler kernels [2].

MSC 2010: 26A33, 33E12, 32A17
Keywords: Nonlocality, fractional calculus, Mittag-Leffler kernels

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A conjecture on the spectral radius of a bipartite graph

YEN-JEN CHENG[1], CHIH-WEN WENG[2], FENG-LEI FAN[3]

[1,2]Department of Applied Mathematics, National Chiao Tung University, Taiwan
[3]Department of Photonics, Harbin Institute of Technology, China

emails: [1]yjc7755.am01g@nctu.edu.tw; [2]weng@math.nctu.edu.tw

The spectral radius of a graph is the maximum eigenvalue of its adjacency matrix. Let G be a bipartite graph with e edges and without isolated vertices. It was known that the spectral radius of G is at most the square root of e, and the upper bound is attained if and only if G is a complete bipartite graph. Bhattacharya, Friedland and Peled [1] conjectured that a non-complete bipartite graph which has the maximum spectral radius with given e and bi-order (p, q) is obtained from a complete bipartite graph by adding one vertex and a corresponding number of edges. We find a counter example of this conjecture. Under the additional assumption that e at least $pq - q$ or p at most 5, where p at most q, we prove a weaker version of the above conjecture that drops the non-complete assumption of the bipartite graph G. Our method is based on a new tight upper bound of the spectral radii of bipartite graphs with bi-order (p, q) and a prescribed degree sequence of the part of order p.

MSC 2010: 05C50, 15A18, 05C35, 15A42.
Keywords: Adjacency matrix, bipartite graph, degree sequence, spectral radius

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Stability of Caputo non-instantaneous impulsive fractional differential equations with delays

SNEZHANA HRISTOVA[1], RAVI AGARWAL[2,3], RONALD O'REGAN[4]

[1]Plovdiv University, Plovdiv, Bulgaria
[2]Texas A&M University-Kingsville, Kingsville, TX 78363, USA
[3]Florida Institute of Technology, Melbourne, FL 32901, USA
[4]National University of Ireland, Galway, Ireland

emails: [1]snehri@gmail.com; [2]agarwal@tamuk.edu; [3]donal.oregan@nuigalway.ie

Impulsive differential equations arise from real world problems to describe the dynamics of processes in which sudden, discontinuous jumps occur. Such processes are natural in biology, physics, engineering, etc. In the literature there are two popular types of impulses:

- *instantaneous impulses*- the duration of these changes is relatively short compared to the overall duration of the whole process;
- *non-instantaneous impulses* - an impulsive action, which starts abruptly at a fixed point and its action continues on a finite time interval.

In this talk Caputo fractional differential equations with non-instantaneous impulses and bounded delays are studied. Both basic approaches in the interpretation of the solutions of the fractional equation deeply connected with the presence of non-instantaneous impulses are discussed and illustrated on several examples.

There are several approaches in the literature to study stability, one of which is the Lyapunov approach. Some difficulties have been encountered when one applies the Lyapunov technique to Caputo fractional differential equations. The basic question which arises is the definition of the derivative of the Lyapunov like function along the given fractional equation. Initially a brief overview of the basic fractional derivatives of Lyapunov functions used in the literature is given and their advantages/disadvantages are discussed and illustrated on examples. Lyapunov functions and Razumikhin technique are applied to study stability properties of Caputo fractional differential equations with non-instantaneous impulses and bounded delays. Comparison results using this definition and scalar fractional differential equations are presented and several sufficient conditions for stability, uniform stability, asymptotic stability, Mittag-Leffler stability are established. Several examples are given to illustrate the theory. Also some applications to neural networks with bounded delays and impulsive perturbations acting as non-instantaneous impulses are presented.

MSC 2010: 34A08, 34K37, 34A37, 34K20
Keywords: Caputo fractional derivative, non-instantaneous impulses, Lyapunov functions, Caputo fractional Dini derivative, Razhumikhin method
Acknowledgement: Research was partially supported by the Fund NPD, Plovdiv University, No. FP17-FMI-008.

Generalized Reimann calculus on fractal sets and curves and application

ALIREZA KHALILI GOLMANKHANEH

Islamic Azad University, Urmia, Iran

emails: alirezakhalili2002@yahoo.co.in

In this paper, we review the fractal calculus and basic concepts. The Cantor-like sets are considered as the support of functions and their derivatives and integrals are given. The generalized Cantor cubes are presented and fractal calculus is extended on them. As the applications of formalism, the super-, sub-, and normal-diffusion are characterized. More, the mathematical models are suggested for the fractal grating in optic and fractal space-time in quantum and classical mechanics.

MSC 2010: 81Q35, 28A80, 76M60
Keywords: Cantor sets, fractal calculus

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Proximal algorithm for compressive sensing

MEHRDAD LAKESTANI

University of Tabriz, Tabriz, Iran

email: lakestani@tabrizu.ac.ir

Proximal algorithm can be used for solving non-smooth, constrained and large-scale optimization problems [3]. Thus it can be used successfully for image processing purposes [1]. Let $A \in \mathbb{R}^{m\times n}$ with $m < n$ or $m \ll n$ (in compressed sensing), $b \in \mathbb{R}^m$, and $x \in \mathbb{R}^n$. A basis pursuit problem is a constrained minimization problem as follows:

$$\min_{x\in\mathbb{R}^n} \{||x||_1 : \quad Ax = b\}, \tag{1}$$

which gives the solution of the under determined linear system $Ax = b$ with minimal L_1 norm. In this paper we improve and use proximal gradient algorithm to solve basis pursuit and related sparse optimization problems [4, 2].

MSC 2010: 65K10, 49M99, 90C51
Keywords: Proximal algorithms, Compressed sensing, L_1 minimization, Basis pursuit

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Review of the Tau method

SEDAGHAT SHAHMORAD[1], YOUNES TALAEI[2]

[1,2]University of Tabriz, Tabriz, Iran.

emails: [1]shahmorad@tabrizu.ac.ir; [2]y_talaei@tabrizu.ac.ir;

The Tau method, introduced for the first time by C. Lanczos [1] in 1938, in order to find approximate solution for some physical problems. In 1956, C. Lanczos and E. L. Ortiz introduced the recursive approach of the Tau method based on Canonical polynomials for numerical study of ordinary differential equation of the form

$$\sum_{i=0}^{\nu} p_i(x)y^{(i)}(x) = f(x), \quad g_j(y) = d_j, \qquad j = 1, ..., \nu \quad a \leq x \leq b, \tag{1}$$

where $p_i(x), f(x)$ are polynomials of finite degree and g_j are some linear functionals acting on $y(x)$ [2, 3]. The Operational approach of the Tau method is based on three simple operational matrices, that introduced for the first time in 1981 by E. L. Ortiz and H. Samara for numerical solution of nonlinear ordinary differential equations [4] and in 2002 it was extend by M. Hosseini and S. Shahmorad for numerical solution of linear integro-differential equations ([5]).

MSC 2010:
Keywords: The Tau method, polynomial solutions, Matrix formulation.

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Stability of linear switched impulsive systems with unstable subsystems

VITALIĬ I. SLYN'KO[1], CEMİL TUNÇ[2]

[1] S. P. Timoshenko Institute of Mechanics of NAS of Ukraine, Kiev, Ukraine
[2] Van Yuzuncu Yil University, Van, Turkey

emails: [1]vitstab@ukr.net; [2]cemtunc@yahoo.com

We consider a Cauchy problem for a linear switched impulsive system [1]

$$\begin{aligned} \frac{dx}{dt} &= A_{\sigma(t)}x(t), \quad t \in (\tau_k, \tau_{k+1}), \\ \Delta x(t) &= B_{\sigma(t)}x(t), \quad t = \tau_k, \quad x(t_0) = x_0, \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$, $\sigma(t)$ is a left continuous piecewise constant function which values belonging to the finite set $\{1, \ldots, N\}$, $A_m \in \mathbb{R}^{n\times n}$, $m = 1, \ldots, N$, $\{\tau_k\}_{k=1}^{\infty} \subset \mathbb{R}$ is an increasing sequence of moments of impulsive action having a single concentration point at infinity, $t_0 < \tau_1$, $x_0 \in \mathbb{R}^n$, $\Delta x(t) = x(t+0) - x(t)$, $B_m \in \mathbb{R}^{n\times n}$, $m = 1, \ldots, N$.

Let us define the structural sets of the linear impulsive system $\mathcal{A} = \{A_1, \ldots, A_N\}$, $\mathcal{B} = \{B_1, \ldots, B_N\}$. With each pair of matrices $(A_m, B_m) \in \mathcal{A}\times\mathcal{B}$ we will associate a positive number θ_m (residence time), so if $\sigma(\tau_k) = m$, then $\tau_{k+1} - \tau_k = \theta_m$. The triple (A_m, B_m, θ_m) defines the subsystem of the hybrid system (1)

$$\begin{aligned} \frac{dz}{dt} &= A_m z(t), \quad t \neq k\theta_m, \\ \Delta z(t) &= B_m z(t), \quad t = k\theta_m, \quad z(t_0) = z_0. \end{aligned} \tag{2}$$

Note that the system (1) is not assumed to be periodic, so Floquet's theory is not applicable in this case.

We propose a new method for studying the stability of the hybrid system (1) for the case when all the matrices of the set $\mathcal{A}$ do not satisfy the Routh-Hurwitz condition, matrices from $\mathcal{B}$ do not satisfy the Schur's condition, and the subsystems (2) are all unstable. The proposed method of investigation is based on the ideas of commutator calculus [2].

MSC 2010: 93D21, 34D20, 34A38, 34A3.
Keywords: Lyapunov's direct method, switched systems, impulsive systems.
Acknowledgement: This work was partially supported by the Ministry of Education and Science of Ukraine project 0116U004691.

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Quaternions and their applications

YUSUF YAYLI

Ankara University, Ankara, Turkey

email: yayli@science.ankara.edu.tr

Quaternions are brother of complex numbers. Quaternions defined in four dimensional space have quite good applications in three-dimensional kinematics field. In the modeling of robot movements, quaternions comes out as a screw operator and a rotation operator. Another application field is the linear spherical interpolation. Quaternionic fractals are also topics that have been studied in recent years. In quantum physics, they facilitate for the construction of unitary matrices $SU(2)$.

This talk will be about the historical development of quaternions and their application to various fields.

MSC 2010: 11R52, 15B33, 70B10, 70E15.
Keywords: Quaternion, split quaternion, interpolation

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Abstracts of participants' talks

Existence of positive solutions for a singular fractional boundary value problem

ASGHAR AHMADKHANLU[1], VEDAT SUAT ERTURK[2]

[1]Azarbaijan Shahid Madani University, Tabriz, Iran
[2]Ondokuz Mayis University, Samsun, Turkey

emails: [1]ahmadkhanlu@azaruniv.ac.ir; [2]vsetrurk@omu.edu.tr;

Fractional derivative, as an extension of ordinary derivative, is a suitable tool for modeling of various physical phenomena, chemical processes and engineering.

Furthermore, fractional calculus has been found many applications in classical mechanics and the calculus of variations, and is a very useful means for obtaining solutions of nonhomogenous linear ordinary and partial differential equations.

The study of the existence and uniqueness of solution or multiplicity of solutions of initial and boundary value problem, including fractional differential equations, has a lot of importance in theoretical arguments. Remarkable researches in concern with the existence and multiplicity of positive solutions for nonlinear fractional boundary value problems have been done using fixed point theorems up to now (see [1, 3, 4]).
The aim of this work is to study the fractional differential equation

$$^cD_0^q u(t) + \lambda f(t, u(t), u'(t)) = 0 \qquad 1 < q \leq 2, \quad t \in J = [0, 1]$$

where $^cD_0^\alpha$ is the Caputo fractional derivetive, subject to the boundary conditions

$$u'(0) = \lambda u(0) \quad u'(1) = 0.$$

Sufficient and necessary conditions will be presented for the existence and uniqueness of solution of this fractional boundary value problem.

MSC 2010: 34A08, 34B18, 26A33
Keywords: Boundary value problem, fractional derivative, fixed point theorem,

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Strongly graded rings over hereditary Noetherian prime rings

EVRİM AKALAN[1], HIDETOSHI MARUBAYASHI[2], AKIRA UEDA[3]

[1] Hacettepe University, Ankara, Turkey
[2]Naruto University of Education, Tokushima, Japan
[3]Shimane University, Shimane, Japan

email: [1]eakalan@hacettepe.edu.tr

Let $R = \bigoplus_{n\in\mathbb{Z}} R_n$ be a strongly graded ring of type $\mathbb{Z}$ where R_0 is a hereditary Noetherian prime ring. In this paper, we investigate and completely describe the structure of projective ideals of R and prove that R is a strongly G-HNP ring.

MSC 2010: 16W50,16D40, 16A18
Keywords: Strongly graded ring, hereditary Noetherian prime ring, projective ideal

Analysis of behaviors of solutions of a coupled Volterra integro-differential equations

IREM AKBULUT[1], CEMİL TUNÇ[2]

[1] Siirt University, Siirt, Turkey
[2] Van Yuzuncu Yil University, Van, Turkey

emails: [1]iremmmatematik@gmail.com; [2]cemtunc@yahoo.com

In this paper, we discuss the existence of periodic solutions for a coupled nonlinear Volterra integro-differential equations with delay. We use some theorems of fixed point theory to prove the result to be given here. We compare our results with that can be found in the literature.

MSC 2010: 45D05, 45M10, 45Jxx
Keywords: Volterra integro-differential equations, coupled equations, periodic solution, fixed point theorem

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Modified expansion function method to the nonlinear problem

TOLGA AKTURK[1], HASAN BULUT[2]

[1]Ordu University, Ordu, Turkey
[2]Firat University, Elazig, Turkey

emails: [1]tolgaakturk@odu.edu.tr; [2]hbulut@firat.edu.tr

In this article, the solutions of the DSW equation are obtained by using the modified expansion function method. Real and imaginary solutions are obtained according to the coefficients obtained from algebraic equation systems. Two and three dimensional graphics of the found solutions are drawn with the Mathematica program by selecting the appropriate parameters.

Drinfel'd-Sokolov-Wilson equation system,

$$u_t + pvv_x = 0, \tag{1}$$

$$v_t + qv_{xxx} + ruv_x + su_xv = 0. \tag{2}$$

MSC 2010: 35C07, 35C08, 35J60
Keywords: The modified expansion function method (MEFM), Drinfel'd-Sokolov-Wilson Equation (DSW), the soliton solutions

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Some properties of conformal generic submersions

MEHMET AKİF AKYOL

Bingöl University, Bingöl, Turkey

email: mehmetakifakyol@bingol.edu.tr

Akyol and Şahin [2] introduced the notion of conformal semi-invariant submersions from almost Hermitian manifolds. The present talk deal with the study of conformal generic submersions from almost Hermitian manifolds which extends semi-invariant submersions, generic Riemannian submersions and conformal semi-invariant submersions a natural way. We mention some examples of such maps and obtain characterizations and investigate some properties, including the integrability of distributions, the geometry of foliations and totally geodesic foliations. Moreover, we obtain some conditions for such submersions to be totally geodesic and harmonic, respectively.

MSC 2010: 53C43, 53C20
Keywords: Kähler manifold, Riemannian submersion, generic Riemannian submersion, conformal submersion, conformal generic submersion, vertical distribution.

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Approximate confidence interval based on winsorized mean for the coefficient of variation of positively skewed populations

HAYRİYE ESRA AKYÜZ

Bitlis Eren University, Bitlis, Turkey

email: heakyuz@beu.edu.tr

The coefficient of variation (CV), as an important measure of variation, has been used in many fields such as medicine, biology, physics, finance, toxicology, business, engineering, life insurance and survival analysis. It is free from the unit of measurement and it can be used for comparing the variability of two different populations. In this study are proposed a confidence interval based on winsorized mean for the population coefficient of variation in the skewed distributions. This confidence interval is based on the Bonett (2006) formula which calculates an confidence interval for the standard deviation of non-normal distributions. A simulation study was made to compare this confidence interval and existing confidence intervals in terms of the coverage probability and average width for normal and some skewed distributions. The number of simulation replications is M = 50.000 for each case and sample size is used as n = 15, 25, 50, 100 using the program written in MATLAB R2016a. Simulation study showed that the coverage probabilities of the proposed confidence interval were very close to nominal confidence level. In addition to, proposed approximate confidence interval also performed well in terms of average width. Wider confidence intervals were obtained for large values of the population coefficient of variation. As a result; it is recommended to use the approximate confidence interval for the coefficient of variation of positively skewed populations.

MSC 2010: 62F10, 62F12, 62F35
Keywords: Average width, coefficient of variation, coverage probability, winsorized mean

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Numerical study of nanofluids under DDMC in a lid driven cavity

NAGEHAN ALSOY-AKGÜN

Van Yüzüncü Yıl University, Van, Turkey

email: nagehanalsoyakgun@yyu.edu.tr

In this study the advantages of nanofluids on double diffusive mixed convection (DDMC) in a lid-driven cavity is analyzed by solving the velocity-vorticity form of the governing equations along with the energy and concentration equations. Numerical computations are conducted using the dual reciprocity boundary element method (DRBEM). Vorticity transport, energy and concentration equations are transformed to the form of modified Helmholtz equations by discretizing the time derivative terms first. The effects of Reynolds number (Re), Richardson numbers (Ri) and buoyancy ratio (N) for variation in volume fraction from 0 to 0.2 is presented for copper based nanofluid graphically and obtained results are good agreement with the results in [1].

MSC 2010:
Keywords: DRBEM, Mixed Convection, Nanofluid, Thermo-solutal buoyancy forces, Lid-driven

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Some properties of sequence class $S^{\beta}(\Delta, F, f)$ defined by a modulus function

HIFSI ALTINOK[1], DERYA DENİZ[2], MİTHAT KASAP[3]

[1,2]Fırat University, Elazığ, TURKEY
[3]Şırnak University, Şırnak, TURKEY

emails: [1]hifsialtinok@gmail.com; [2]deryadeniz485@yandex.com; [3]fdd_mithat@hotmail.com

Aizpuru *et al.* [1] defined the $f-$density of the subset A of $\mathbb{N}$ by using an unbounded modulus function. After then, Bhardwaj [4] introduced $f-$statistical convergence of order α with respect to a modulus function f for real sequences. In the present paper, we define the sequence class $S^{\beta}(\Delta, F, f)$ for $0 < \beta \leq 1$, where f is an unbounded modulus function, Δ is a difference operator in sequences of fuzzy numbers and give some inclusion theorems between $S^{\beta}(\Delta, F, f)$ and the classical sequence classes.

MSC 2010: 40A05, 40A25, 40A30, 40C05, 03E72
Keywords: Statistical convergence, sequence of fuzzy numbers, modulus function, difference operator

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Strongly Cesaro summability of order β with respect to a modulus function

HIFSI ALTINOK[1], MİTHAT KASAP[2], DERYA DENİZ[3]

[1,3]Fırat University, Elazığ, TURKEY
[2]Şırnak University, Şırnak, TURKEY

emails: [1]hifsialtinok@gmail.com; [2]fdd_mithat@hotmail.com; [3]deryadeniz485@yandex.com

In this study, we generalize and examine the sequence classes $w^{\beta}(F, f)$, $w^{\beta,0}(F, f)$ and $w^{\beta,\infty}(F, f)$, where f is an unbounded modulus function and $\beta \in (0, 1]$ is a real number, for sequences of fuzzy numbers and examine some inclusion relations between them.

MSC 2010: 40A05, 40A25, 40A30, 40C05, 03E72
Keywords: Sequence of fuzzy numbers, statistical convergence, modulus function, Cesàro summability

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On eccentricity based indices of generalized Petersen graphs

MEHMET ŞERÍF ALDEMÍR[1], ABDALLA KHDIR ABDALLA MANGURI[2]

[1]Van Yuzuncu Yil University, Van, Turkey
[2] University of Sulaimani, Sulaymaniyah, Iraq

emails: [1]msaldemir@yyu.edu.tr; [2]abdullah6nasim@gmail.com

In this study, we firstly calculate the eccentric connectivity and connective eccentricity indices for the generalized Petersen graphs.

MSC 2010: 05C12
Keywords: Eccentric connectivity index, Connective ecentricity index, Generalized Petersen graphs

On stratified domination and Zagreb indices

MEHMET ŞERÍF ALDEMÍR[1], ABDALLA KHDIR ABDALLA MANGURI[2]

[1]Van Yuzuncu Yil University, Van, Turkey
[2] University of Sulaimani, Sulaymaniyah, Iraq

emails: [1]abdullah6nasim@gmail.com, [2]msaldemir@yyu.edu.tr

In this study, we firstly investigate the relationship between stratified domination number and Zagreb indices.

MSC 2010: 05C12
Keywords: The first Zagreb index, The second Zagreb index, Stratified domination

Inference algorithms for jump-diffusion approximations of multi-scale processes

DERYA ALTINTAN[1], HEINZ KOEPPL[2]

[1]Department of Mathematics, Selçuk University, Konya, Turkey
[2]Department of Electrical Engineering and Information Technology, Technische Universität Darmstadt, Darmstadt, Germany

emails: [1]altintan@selcuk.edu.tr; [2]heinz.koeppl@bcs.tu-darmstadt.de

In a biochemical system, the abundance of molecular species and the magnitude of reaction rates can vary in a wide range. This diversity leads to hybrid models which combine deterministic and stochastic modeling approaches. We proposed a jump-diffusion approximation to model biochemical processes with multi-scale nature [3]. The idea of the model is to partition reactions into fast, slow groups and to combine Markov chain updating scheme for the slow set with a diffusion (Langevin) approach updating scheme for the fast set. Based on the state vector representation of the jump-diffusion approximation which is defined as a summation of the random time change model and the Langevin equation, we proved that the joint probability density function of jump-diffusion approximation satisfies the hybrid master equation which is the summation of the corresponding chemical master equation and the Fokker-Planck equation [4].

In this study, we develop an inference algorithm to estimate the hidden states/parameters of reaction systems whose posterior distribution satisfies the hybrid master equation. To construct the algorithm, we combine particle filtering/smoothing methods [2] with Gibbs Monte Carlo Markov Chain scheme [1]. To illustrate the method, we implement the algorithm to biochemical processes.

MSC 2010: 60H30, 60J28, 92B05
Keywords: Deterministic modeling, stochastic modeling, diffusion (Langevin) approach, jump-diffusion approximation, chemical master equation, Fokker-Planck equation, Gibbs Monte Carlo Markov Chain
Acknowledgement: This work is supported by the Scientific and Technological Research Council of Turkey (TÜBİTAK) Program no:3501 Grant, no. 115E252.

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On the global exponential stability of nonlinear neutral differential equations with time-varying delays

YENER ALTUN[1], CEMİL TUNÇ[2]

[1,2]Van Yüzüncü Yıl University, Van, Turkey

emails: [1]yener-altun@yahoo.com; [2]cemtunc@yahoo.com;

In this study, we investigated the global exponential stability of the zero solution of a neutral differential equation with time-lags. We find sufficient conditions which guarantee the global exponential stability of the zero solution of the equation. We benefit from the Lyapunov functional as a basic tool and the obtained result includes and improves some ones in the literature. An example is given to illustrate the applicability and correctness of the obtained result by MATLAB-Simulink.

MSC 2010: 34K20, 34K40 93D05.
Keywords: Neutral differential equation, global exponential stability, Lyapunov functional, matrix inequality, multiple delays.

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On the exponential stability in nonlinear neutral differential equations

YENER ALTUN[1], CEMİL TUNÇ[2], ABDULLAH YİĞİT[3]

[1,2,3] Van Yüzüncü Yıl University, Van, Turkey

emails: [1]yeneraltun@yyu.edu.tr; [2]cemtunc@yahoo.com; [3]a-yigit63@hotmail.com

In this work, we consider a nonlinear time-varying delay system of neutral equations with periodic coefficients in the form

$$\frac{d}{dt}(y(t) + Dy(t-\tau(t))) = A(t)y(t) + B(t)y(t-\tau(t))) + F(t, y(t), y(t-\tau(t)))$$

where

$$\|F(t,u,v)\| \leq q_1\|u\|^{1+w_1} + q_2\|v\|^{1+w_2}, \qquad q_1, q_2, w_1, w_2 > 0$$

We obtain some new estimates characterizing the exponential decay of solutions at infinity and the attraction sets of the zero solution.

MSC 2010: 34K20, 34K40
Keywords: Neutral equation, Lyapunov- Krasovskii functional.

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On a new type of q-Baskakov-Kantorovich operators

NAZLIM DENIZ ARAL[1], ZEYNEP SEVİNÇ[2]

[1,2] Bitlis Eren University, Bitlis, Turkey

emails: [1]ndaral@beu.edu.tr; [2]z.sevinc13@gmail.com

In this work, we have introduced a new type of q-analogous of Baskakov-Kantorovich operators and investigated their statistical approximation properties. By using a weighted modulus of smoothness, we have given some direct estimations for error in the case $0 < q < 1$.

MSC 2010: 41A36, 41A30, 41A25
Keywords: q-analysis, q-Baskakov-Kantorovich operators

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Second-order difference approximation for nonlocal boundary value problem with boundary layers

DERYA ARSLAN

Ministry of National Education, Bitlis, Turkey

email: ayredlanu@gmail.com

We consider uniform finite difference method on Bakhvalov mesh for a linear singularly perturbed multi-point boundary value problem

$$\begin{aligned} &\varepsilon^2 u''(x) + \varepsilon a\,(x)\,u'\,(x) - b(x)u(x) = f\,(x)\,, && 0 < x < 1, \\ &u\,(0) = A, && \\ &u(1) - \gamma u(l_1) = B, && 0 < l_1 < 1, \end{aligned}$$

where $0 < \varepsilon << 1$ is a small perturbation parameter; A, B and γ are given constants; $a(x) \geq 0$ and $b\,(x) \geq \beta > 0$; $f\,(x)$ and $a\,(x)$ are assumed to be sufficiently continuously differentiable functions in $[0, 1]$. Morever the solution $u(x) = 0$ and $u(x) = 1$. In this study, we give asymptotic properties of the exact solution. We discretize the problem on a nonuniform mesh and obtain finite difference scheme. Finally, error estimation showed that the proposed method is the second-order uniform convergent independently of the perturbed term ε in the dicrete maximoum norm.

MSC 2010: 65L10, 65L11, 65L12, 65L15, 65L20, 65L70, 34B10
Keywords: Singular perturbation, finite difference scheme, Bakhvalov mesh, second-order convergence

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Approximation by bivariate Bernstein-Kantorovich operators on a triangular domain

REŞAT ASLAN[1], AYDIN İZGİ[2]

[1,2] Harran University, Şanlıurfa, Turkey

emails: [1]resat.aslan@iskur.gov.tr; [2]a_izgi@harran.edu.tr

The aim of this paper is to study the convergence and approximation properties of the bivariate Bernstein-Kantorovich operators on a triangular domain.Our operator is defined as below

$$R_n(f;x.y) = \sum_{k=0}^{n}\sum_{l=0}^{n-k}\varphi_{n,k,l}(x,y)\left(\frac{n+1}{2}\right)^2 \int\limits_{2\frac{k}{n+1}-1}^{2\frac{k+1}{n+1}-1}\int\limits_{2\frac{l}{n+1}-1}^{2\frac{l+1}{n+1}-1} f(s,t)dsdt$$

where

$$\varphi_{n,k,l}(x,y) = \binom{n}{k}\binom{n-k}{l}\left(\frac{1+x}{2}\right)^k\left(\frac{1+y}{2}\right)^l\left(1-\frac{1+x}{2}-\frac{1+y}{2}\right)^{n-k-l}$$

The approximation properties were researched and approximation degree of this operator by means of the partial and complete modulus of continuity on the triangular domain were investigated and numerical examples were given and graphics were drawn by Mapple programme.We estimate the order of approximation by Voronovskaja type result and also demonstrate the convergence of the operators R_n to a certain function and the comparison of the convergence of the bivariate Bernstein-Kantorovich operators to the function through illustrations using .

MSC 2010: 41A36, 41A35, 41A10
Keywords: Approximation theory, linear positive operators,partial and complete modulus of continuity, bernstein-kantorovich operators, degree of approximation

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Synchronization control of two chaotic systems via a novel fuzzy control method

ÖZKAN ATAN[1], FATİH KUTLU[2]

[1,2]Van Yüzüncü Yıl University, Van, Turkey
emails: [1]oatan@yyu.edu.tr; [2]fatihkutlu@yyu.edu.tr

In this study, synchronization control will be performed using intiutionistic fuzzy method for synchronization of two chaotic systems. The stability range of the system will be determined by the Lyapunov method and synchronization control will be provided according to this method. The designed synchronization method is compared with the studies in the literature.

MSC 2010: 93C42, 34C28, 94D05
Keywords: Chaotic systems; chaotic synchronization; fuzzy control, intuitionistic fuzzy sets.

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On the structure of Ricci solitons on gradient Einstein-type manifolds

MERVE ATASEVER[1], SEZGİN ALTAY DEMİRBAĞ[2]

[1,2]İstanbul Technical University, İstanbul, Turkey

emails: [1]atasever17@itu.edu.tr; [2]saltay@itu.edu.tr

There has been increasing interest especially on the study of Einstein manifolds and their several generalizations in Riemannian geometry. We say that (M, g) is a gradient Einstein-type manifold if we have

$$\alpha Ric + \beta Hess(f) + \mu df \otimes df = (\rho r + \lambda)g, \tag{1}$$

for some $\alpha, \beta, \mu, \rho \in \mathbb{R}$, and $f \in C^{\infty}(M)$. In this paper we study the notion of gradient Einstein-type structure on a Riemannian manifolds such as gradient Ricci solitons and quasi-Einstein manifolds. Then,some examples for this kind of manifolds will be given in the following part of the paper.

MSC 2010: 53B15,53B20,53C21,53C25
Keywords: Ricci solitons, Gradient Einstein-type manifolds, Quasi-Einstein manifolds, Parallel vector fields, Warped products

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A non-local model for E and N cadherin-dependent cell-cell adhesion

EMINE ATICI ENDES[1], JONATHAN A. SHERRATT[2]

[1,2] Heriot-Watt University, Edinburgh, UK

emails: [1]eae2@hw.ac.uk; [2]j.a.sherratt@hw.ac.uk

Cell adhesion molecules, such as Epithelial (E)-cadherin and Neural (N)-cadherin, have an essential importance on the binding of one cell to another at the cell surface. Known also as cell-cell adhesion this process plays a critical role on tissue formation during early embryo development, immune responses and wound healing. All these biological functions require the coordinated movement of cells in particular ways to specific locations. However; it is not clear how cell junctions control this coordinated migration. Due to this fact, we examine throughly the role of E- and N-cadherins on cell migration phenomenon. In other words, we investigate how E- and N-cadherins affect cell direction during migration. In order to understand their impacts on the direction of cell movement, we develop a new continuous mathematical model consisting of two different direction functions that represent E cadherin and N cadherin, respectively.

The model with non-local adhesion term:

$$\overbrace{\frac{\partial u}{\partial t}}^{\substack{change\ in\\ cell\ density}} = \overbrace{D\frac{\partial^2 u}{\partial x^2}}^{\substack{cell\\ diffusion}} - \overbrace{\frac{\partial}{\partial x}(uK(u))}^{\substack{adhesion\\ force}} \quad \text{(Mass Conservation Equation)}$$

where

$$K(u) = \frac{\phi}{R}\int_{-R}^{R} \alpha\Big(g_1(u(x+x_0))w_1(x_0) + g_2(u(x+x_0))w_2(x_0)\Big)dx_0$$

K(u) is the non-local term, α reflects the strentgh of adhesion force, $g_1(u(x+x_0))$ and $g_2(u(x+x_0))$ are the nature of the forces, and $w_1(x_0)$ and $w_2(x_0)$ describe the direction and magnitude of the force between cells.

MSC 2010: 35R09, 35C07
Keywords: Cell-cell adhesion, Integro-PDE model, E and N-cadherins
Acknowledgement: This research is supported by The Ministry of National Education a Centre for Postgraduate Training (YLSY Program) funded by The Turkish Government, Ankara, Turkey

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Approximating the stochastic evolution via difference equations

OZGUR AYDOGMUS

Social Sciences University of Ankara, Ankara, Turkey

email: ozgur.aydogmus@asbu.edu.tr

We consider a chain-binomial process modeling the evolution of behavioral traits in a population. Mean field equations of the model are found and analyzed. The behavior of the chain-binomial process is probabilistically linked to the mean field equation. As a result of this link, we are able to show that the mean fixation time is an exponentially increasing function of time if there exist an interior evolutionary stable state. We also present simulation results for the process to validate our analytical findings.

MSC 2010: 91A22, 60J10, 39A30
Keywords: evolutionary game theory, difference equations, chain-binomial processes

Reduction and coreduction of modules

JAFAR A'ZAMI[1], MARYAM KHAJEPOUR[2]

[1,2]University of Mohaghegh Ardabili, Ardabil, Iran

emails: [1]jafar.azami@gmail.com; [2]maryamkhajepour@uma.ac.ir

Throughout this paper, all rings are commutative rings with identity and all modules are unital. Let R be a ring and M be an R-module and N, K be submodules of M.The product of N and K is defined as $NK = (N : M)(K : M)M$. (see [4]). Let M be an R-module and N, K be submodules of M. We say that K is a reduction of N, if $K \subseteq N$ and there exists a natural number s such that $KN^s = N^{s+1}$. Let M be an R-module and N, K be submodules of M such that $N \subseteq K$. We say that K is a coreduction of N, if there exists a natural number s such that $(0 :_M Ann(K)Ann^s(N)) = (0 :_M Ann^{s+1}(N))$. We denote it by $C(KN^s) = C(N^{s+1})$. In this paper, we prove some relations about this notions over multiplication and comultiplication modules.

MSC 2010: 13C13, 13E15
Keywords: Reduction, integral closure, multiplication modules, integrally dependent

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Roller coaster surface according to modified orthogonal frame in Euclidean space

SELÇUK BAŞ[1], TALAT KÖRPINAR[2], RIDVAN CEM DEMİRKOL[3], MUSTAFA YENEROĞLU[4]

[1,2,3]Muş Alparslan University, Muş, Turkey
[4]Fırat University, Elazığ, Turkey

emails: [1]selcukbas79@hotmail.com; [2]talatkorpinar@gmail.com; [3]rcdemirkol@gmail.com; [4]mustafayeneroglu@gmail.com

In this study, Roller Coaster surfaces according to modified orthogonal frame is introduced in Euclidean space 3-space. The Gaussian curvature, mean curvature, first and second fundamental form of coefficients of Roller Coaster surfaces of are examined. Then, we obtain some characterizations of Roller Coaster surfaces in the Euclidean space 3-space.

MSC 2000: 53A04, 53A05
Keywords: Modified orthogonal frame, roller coaster surfaces, Gaussian curvature, mean curvature

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On mechanisms in three-dimensional Minkowski space

ŞENAY BAYDAŞ[1], BÜLENT KARAKAŞ[2]

[1,2]Van Yüzüncü Yıl University, Van, Turkey

emails: [1]senaybaydas@gmail.com; [2]bulentkarakas@gmail.com;

In Euclidean space a mechanism is designed by Denavit-Hartenberg representation. In the Minkowski 3-space, the rotations about the standard spacelike coordinate axes through the hyperbolic angle are represented with the orthonormal matrices. In this paper, a mechanism is designed in threedimensional Minkowski space with D-H parameters.

MSC 2010: B3B30, 70B10
Keywords: Denavit-Hertenberg parameters,Lorentzian geometry, space-like vector

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Groups whose codegree graphs have no triangle

HOSHANG BEHRAVESH[1], MEHDI GHAFFARZADEH[2], MOHSEN GHASEMI[3]

[1,3] Urmia University, Urmia, Iran
[2] Islamic Azad University, Khoy, Iran

emails: [1]h.behravesh@urmia.ac.ir; [2]ghaffarzadeh@iaukhoy.ac.ir; [3]m.ghasemi@urmia.ac.ir

For a character χ of a finite group G, the number $\mathrm{cod}(\chi) = |G : \mathrm{Ker}(\chi)|/\chi(1)$ is called the codegree of χ. The codegree graph $\Gamma(G)$ is a graph whose vertex set is the all primes dividing some codegree of a character of G and there is an edge between two distinct primes p and q if pq divides some codegree of a character of G. In this paper, we show that if G is a finite group whose codegree graph has no triangle then $\Gamma(G)$ has at most 5 vertices.

MSC 2010: 20C15, 20D05
Keywords:finite group, irreducible character, codegree graph

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Some algebraic properties of elliptic biquaternions

MURAT BEKAR[1], YUSUF YAYLI[2]

[1]Gazi University, Polatli/Ankara, Turkey
[2]Ankara University, Ankara, Turkey

emails: [1]muratbekar@gazi.edu.tr; [2]yayli@science.ankara.edu.tr

In this study, firstly, we give a brief summary of the concepts elliptic numbers, real quaternions and complex quaternions. Afterwards, we consider the algebra of elliptic biquaternions and give some algebraic properties of this algebra.

MSC 2010: 11R52, 53A17, 53A35
Keywords: Elliptic number, real quaternion, complex quaternion, elliptic biquaternion

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Quasilinearization method in problems of the subcritical deformation of flexible shell systems

ELENA BESPALOVA[1], NATALIIA YAREMCHENKO[2]

[1,2]Institute of mechanics of NAS of Ukraine, Kiev, Ukraine

emails: [1]elena_bespalova@ukr.net; [2]nataliya.petrivna@ukr.net

Compound systems of flexible shells with various geometry and structure represent design schemes of many constructions of modern engineering. Inves-tigation of elastic deformation of such systems necessitates to solve nonlinear boundary-value problems for systems of high-order differential equations, which meets with certain computational difficulties even for advanced computer engineering.

In the report, the technique for determining the stress-strain state of compound shells within the wide range of acting loads up to their limiting critical values is proposed. It combines the Newton-Kantorovich-Raphson linearization method (quasilinearization method) [1] and the orthogonal-sweep method [2]. For the technique, the quadratic convergence of the process of successive approximations and high accuracy in solving linearized problems are typical.

Values of the critical loads are determined using a computational criterion by which $\lambda_{cr}^{lim} \in [\lambda_{n-1}, \lambda_n]$, where λ_{n-1} is the maximum value of the load at which the process monotonically converges, λ_n is its minimum value at which the conditions of monotonic convergence are not fulfilled ($n = 1, 2, ...$ is the step of the iteration process).

Using, as an example, a space-rocket apparatus, it is shown that the process of successive approximations $\varepsilon = \varepsilon(n)$ (ε is a certain chosen charac-teristic) may be of various nature, have different convergence rapidity and contains in a number of cases very shallow segments (plateau) depending on the geometrical and physical features of the object being studied. Such unsuspected character of this process demands enhanced attention in intro-ducing automation in calculations of the subcritical state of shell systems.

Keywords: flexible shell systems, nonlinear boundary-value problems, qua-silinearization method, orthogonal-sweep method, analysis

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MHD natural convection flow in a porous cavity

CANAN BOZKAYA

Middle East Technical University, Ankara, Turkey

email: bcanan@metu.edu.tr

A numerical investigation of natural convection flow in a cavity filled with a fluid-saturated porous medium in the presence of uniform magnetic field is performed. The steady, viscous, incompressible flow inside the porous medium is assumed to obey the Darcy law. The fluid physical properties are constant except the density in the body force term which is treated according to Boussinesq approximation. The fluid and porous medium are in thermal equilibrium. The governing equations subject to appropriate boundary conditions are solved by using the dual reciprocity boundary element method (DRBEM) which transforms the differential equations into equivalent boundary integral equations by treating the non-homogeneity through a radial basis function approximation. A parametric study illustrating the influence of the physical parameters on the flow and heat transfer characteristics is carried out and the results are visualized in terms of the streamlines, isotherms and the average Nusselt number.

MSC 2010: 65N38, 76M15, 76R10, 76W05
Keywords: MHD, cavity, porous medium, DRBEM

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A general approach to find generating sets of certain finite subsemigroups of symmetric inverse semigroup

LEYLA BUGAY

Çukurova University, Adana, Turkey

email: ltanguler@cu.edu.tr

Let I_n be the inverse semigroup consists of all partial bijections on $X_n = \{1, \ldots, n\}$ which is called symmetric inverse semigroup. It is known from Wagner-Preston Theorem that every finite inverse semigroup is isomorphic to a subsemigroup of a suitable symmetric inverse semigroup. Hence the symmetric inverse semigroups and their subsemigroups have an important role in Inverse Semigroup Theory like as the symmetric groups in Group Theory. Let $\alpha \in I_n$. Then α is called *isotone* (*antitone*) if $x < y \Rightarrow x\alpha < y\alpha$ ($x < y \Rightarrow x\alpha > y\alpha$) for $\forall x, y \in \mathrm{dom}(\alpha)$, and α is called *monotone* if α is isotone or antitone. Clearly the set of all monotone partial bijections is a subsemigroup of I_n, denoted by $PODI_n$, and also, for $0 \leq r \leq n-1$, $PODI_{n,r} = \{\alpha \in PODI_n : |\mathrm{im}(\alpha)| \leq r\}$ is a subsemigroup of $PODI_n$. In this talk we give a new and general approach to find any generating set of $PODI_{n,r}$ by using digraphs.

MSC 2010: 20M20
Keywords: Partial bijection, monotone map, generating set
Acknowledgement: My sincere thanks are due to Prof. Dr. Hayrullah Ayık for his helpful suggestions and encouragement.

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On *ve*-degrees in direct and strong products of two graphs

MURAT CANCAN[1], SÜLEYMAN EDÍZ[2], MEHMET ŞERÍF ALDEMÍR[3]

[1,2,3] Van Yuzuncu Yil University, Van, Turkey

emails: [1]mcancan@yyu.edu.tr; [2]suleymanediz@yyu.edu.tr; [3]msaldemir@yyu.edu.tr

Let G be a graph and v be a vertex of G. The ve-degree of the vertex v defined as the number of different edges incident to the vertices of the open neighborhood of v. In this study we investigate the ve-degrees in Cartesian product of two graphs.

MSC 2010: 05C07
Keywords: ve-degree, Cartesian product, Graph operation

Extortion strategies in non-symetric iterated Prisoner's Dilemma

CANSU CENGIZ[1], SERKAN ALI DUZCE[2]

[1,2] Anadolu University, Eskisehir, Turkey

emails: [1]cansucengiz@anadolu.edu.tr; [2]saduzce@anadolu.edu.tr

Good strategies are described for the memory-one symmetric games [1, 2]. Iterated Prisoners Dilemma (IPD) is a symmetric game that players get same payoffs in similar situations with cooperation and defection. The symmetric version of IPD is suitable for evolutionary games, but classic game theory payoffs have to be given in terms of utility functions, which measure the preferences of the players. When interpersonal comparison of utilities is excluded, symmetric game is not suitable. In our study, it is investigated that extortioner player can enforce a different extortionate share in non-symmetric IPD games where players gain different payoffs. Extortioner player gains more than the other player out of their payoffs.

MSC 2010: 91A05, 91A12, 91A20
Keywords: Iterated Prisoner's Dilemma, good Strategy, extortion strategy

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Spectral properties of a q-fractional boundary value problem

F. AYCA CETINKAYA[1], ILKNUR AYDIN[2]

[1,2]Mersin University, Mersin, Turkey

emails: [1]faycacetinkaya@mersin.edu.tr; [2]aydnilknur95@gmail.com

This paper deals with a boundary value problem which is generated by a differential equation with q-Jackson derivative and a discontinuous weight function. The interval, in which the boundary value problem is defined, is finite. By modifying some techniques of [1,2] and [3] we investigate the spectral properties of the above-mentioned boundary value problem.

MSC 2010: 34B08, 34L05
Keywords: q-Jackson derivative, Sturm-Liouville operator, eigenvalues and eigenfunctions

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Mathematical aspects of quantum cryptography

AKRAM CHEHRAZI[1], TURGUT HANOYMAK[2]

[1]Azarbaijan Shahid Madani University, Tabriz, Iran
[2]Van Yüzüncü Yıl University, Van, Turkey

emails: [1]achehrazi95@gmail.com; [2]turguthanoymak@gmail.com;

The integer factorization problem is known to be one of the hardest in mathematics. In modern cryptography, RSA encryption algorithm, which is highly practical and widespread, is based on the integer factorization problem. No efficient classical algorithm for the factorization of large number is known. In 1994, Peter Shor proposed an algorithm for this problem which runs in polynomial time on quantum computer. The scope of this study covers the fundamental concepts of quantum cryptography, quantum computation and the basics of Shor's quantum algorithm.

MSC 2010: 94A60, 94A62, 68P25
Keywords: Superposition principle, qubit, density matrix, Shor's factoring algorithm

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Random process generated by the short incomplete Gauss sums

EMEK DEMIRCI AKARSU

Recep Tayyip Erdogan University, Rize, Turkey

email: emek.akarsu@erdogan.edu.tr

In this talk we investigate a random process generated by the short incomplete Gauss sums and establish an analog of weak invariance principle for these sums. A generalization of the limit distribution of the short incomplete Gauss sums given by theta sums on the metaplectic horocycles is analyzed. This talk is an extension of the author‘s earlier work on the subject. [1]

MSC 2010: 11L05
Keywords: Short Gauss sums; weak invariance principle, random process, metaplectic group.

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A new approach to a bending energy of elastica for space curves in De-Sitter space

RIDVAN CEM DEMİRKOL[1], TALAT KÖRPINAR[2], VEDAT ASİL[3], SELÇUK BAŞ[4]

[1,2,4]Mus Alparslan University, Mus, Turkey
[3]Fırat University, Elazığ, Turkey

emails: [1]rcdemirkol@gmail.com; [2]talatkorpinar@gmail.com; [3]vedatasil@gmail.com; [4]s.bas@alparslan.edu.tr

In this paper, we firstly introduce kinematics properties of a moving particle lying in De-Sitter space S_1^3. We assume that the particle corresponds to a different type of space curves such that they are characterized by using Frenet vector fields in De-Sitter spacetime. Based on this assumption, we present geometrical understanding of the energy on the particle in each Frenet vector fields depending on being a spacelike or timelike curve in S_1^3. Then, we also determine the bending elastic energy functional for the same particle in S_1^3 by assuming the particle has a bending feature of elastica. Finally, we prove that bending energy formula can be represented by the energy on the particle in each Frenet vector field. We conclude our results by providing energy variation sketches with respect to time for different cases.

MSC 2010: 53C41, 53A10
Keywords: Energy, De-Sitter space, Frenet vector fields, elastica

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An analogue of the Artin-Rees Lemma for Artinian modules

İSMAİL HAKKI DENİZLER

Van Yüzüncü Yıl University, Van, TURKEY

email: ismailhd@yyu.edu.tr

In this study we state a useful theorem for Noetherian modules, known as the "Artin-Rees Lemma" after E. Artin and D. Rees who discovered it independently. We will show later how such an Artinian analogue can be deduced from the original result, in a direct and simple way by using Matlis duality[1].

Artin-Rees Lemma: Let R be a Noetherian ring and N a finitely generated R-module (so Noetherian). Let M be a submodule of N, and let I be a proper ideal of R. Then there exists a positive integer c such that for every $n > c$ we have

$$I^n N \cap M = I^{n-c}(I^c N \cap M).$$

(See, for example [2, (8.5)].)

Later we give the analogue result of the Artin-Rees lemma for Artinian modules;

Let A be an Artinian R-module, and let B be a submodule of A. Then for an ideal I of R there exists a non-negative integer c such that

$$B + (0 :_A I^n) = \left(B + (0 :_A I^c) :_A I^{n-c}\right)$$

for all $n \geq c$, $n \in \mathbb{N}$.

We prove this lemma in two steps. First, we consider the special case in which we assume R is a complete semi-local (Noetherian) ring.

Later we complete the proof by supposing that R is an arbitrary (non-trivial commutative) ring. Since A is Artinian, then by [3], there exists a complete semi-local (Noetherian) ring R' such that the module A is, in the natural way, a faithful Artinian R'-module; moreover, a subset of A is an R-module if and only if it is an R'-submodule.

MSC 2010: 13E10, 13E05, 13A15
Keywords: Artinian modules, Noetherian modules, Ideal theory

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Kinematics of 4R and 2RPR mechanisms in Clifford algebra

VEDAT DÖRMA[1], BÜLENT KARAKAŞ[2], ŞENAY BAYDAŞ[3]

[1,2,3]Van Yuzuncu Yil University, Van, Turkey

emails: [1]vedatdorma65@gmail.com; [2]bulentkarakas@gmail.com; [3]senay.baydas@gmail.com

In this paper, Clifford product, Denavit-Hartenberg representation, the forward kinematic equations and forward kinematics of motor algebra are given. Forward kinematics, motor equation and kinematic equations of the 4R and 2RPR mechanisms in Clifford algebra are obtained. Additionally, Matlab program is used for examples.

MSC 2010: 15A66, 68T40, 70B15, 53A17
Keywords: Clifford algebra, Denavit-Hartenberg representation, SCARA robot

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A note on q-Fubini polynomials

UGUR DURAN[1], MEHMET ACIKGOZ[2], SERKAN ARACI[3]

[1]Iskenderun Technical University, Hatay, Turkey
[2]University of Gaziantep, Gaziantep, Turkey
[3]Hasan Kalyoncu University, Gaziantep, Turkey

emails: [1]mtdrnugur@gmail.com; [2]acikgoz@gantep.edu.tr; [3]mtsrkn@hotmail.com

The main aim of this study is to introduce a new extension of Fubini polynomials based on q-numbers. Then, we investigate some of their properties including recurrence relations, differentiate properties and explicit formulas.

MSC 2010: 11B835, 05A19
Keywords: q-numbers, Fubini polynomials, Stirling numbers of the second kind

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On *ve*-degrees in direct and strong products of two graphs

SÜLEYMAN EDÍZ[1], MURAT CANCAN[2], MEHMET ŞERÍF ALDEMÍR[3]

[1,2,3]Van Yuzuncu Yil University, Van, Turkey

emails: [1]mcancan@yyu.edu.tr; [2]suleymanediz@yyu.edu.tr; [3]msaldemir@yyu.edu.tr

Let G be a graph and v be a vertex of G. The *ve*-degree of the vertex v defined as the number of different edges incident to the vertices of the open neighborhood of v. In this study we investigate the *ve*-degrees in direct and strong products of two graphs.

MSC 2010: 05C07
Keywords: ve-degree, direct product, strong product

Statistical analysis of wind speed data with some distributions

NECATİ ERDOGAN[1], ASUMAN YILMAZ[2], MAHMUT KARA[3]

[1,2,3]Van Yuzuncu Yil University, Van, Turkey

emails: [1]nerdogan@yyu.edu.tr; [2]asumanduva@yyu.edu.tr; [3]mkara2581@gmail.com

Precisely and efficiently modeling wind speed is important in estimating the wind energy potential of a specified region. Two- parameter Weibull distribution is the most widely used and accepted distribution in the energy literature. However, it does not model the all wind speed data encountered in nature. Therefore, in this study, different distributions are used for modeling wind energy, such as Gamma, log- normal, Rayleigh. The estimators of the unknown parameters of these distributions are found by using maximum likelihood estimators (MLEs). Finally, hourly wind speed data (m/s), measured hourly at $10m$ from Adilcevaz and anakkale Turkey during December 2017 is used. These data are taken from the Turkish State Meteorological Service. The fit distribution is determined with respect to root mean square error (RMSE), Kolmogorov Smirnov and coefficient of determination R^2 criteria.

MSC 2010: 60J10, 60J22, 11K06
Keywords: : Wind speed, Weibull distribution, Root mean square

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On the existence of periodic solutions of third order nonlinear differential equations with multiple delays

SULTAN ERDUR[1], CEMİL TUNÇ[2]

[1,2]Van Yuzuncu Yil University, Van, Turkey

emails: [1]serdur82@gmail.com; [2]cemtunc@yahoo.com

In this paper, we investigate the existence of periodic solutions of third order nonlinear differential equations with multiple delays by using Lyapunov's second method. We establish sufficient conditions which guarantee the existence of periodic solutions of the considered equations. We give an example to visualize. The obtained results include and improve some results in the literature.

MSC 2010: 37B25, 34C25, 34A34
Keywords: Delay differential equations, Lyapunov method, periodic solution, third order

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The theory of Bézier curves in E^4

ESRA ERKAN[1], SALİM YÜCE[2]

[1,2]Yildiz Technical University, Istanbul, Turkey

emails: [1]eserkan@yildiz.edu.tr; [2]sayuce@yildiz.edu.tr

In this work, we aim to study Bézier curves that is important in Computer Aided Geometric Design and given by the equation, [2]

$$P(t) = \sum_{i=0}^{n} B_{i,n}(t) b_i,\ 0 \leq t \leq 1$$

where the b_i's represent the $n+1$ control points in Euclidean space E^4 with regard to differential geometry. We derive Serret-Frenet elements of a Bézier curve at its all points, starting and ending points in Euclidean space E^4.

MSC 2010: 53A04, 65D07, 68U05
Keywords: Bézier curve, Serret-Frenet frame, triple (ternary) product
Acknowledgement: This work has been supported by the Scientific and Technological Research Council of Turkey (TÜBİTAK).

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Misconceptions regarding representativeness in probability subject of high school students: Van case

ELİF ERTEM AKBAŞ[1], MUSTAFA GÖK[2]

[1,2]Van Yuzuncu Yil University, Van, Turkey

emails: [1]elifertem@yyu.edu.tr; [2]mustafagok@yyu.edu.tr

Probability is a measure of the certainty of an event. Probability, which has an important role in students' preferences in daily life, includes many objects that are difficult to understand. One of these is representativeness, which represents an exemplary situation that could possibly arise as a result of an event. For example, most people think that when tossing a coin, a sequence of six tails the TTTTTT sequence is less probable than the THHTHT sequence. Therefore, in this study, it was aimed to determine the misconceptions in the probability regarding representativeness of high school students (9th, 10th and 11th grade). In the study, the case study, one of the qualitative research methods, was used. The participant of the study constitutes 177 high school students selected by purpose of sampling method. The "Representativeness in Statistical Reasoning: Identifying and Assessing Misconceptions" test, which was developed as a data collection tool [1] and calculated with a reliability of 0.84, was adapted to Turkish. Findings of the study reveal that about half of the students are conceptual misconceptions about the representativeness of probability. It has also been found that as class levels increase, misconceptions about the representativeness of probability decrease.

MSC 2010: 97D70, 97A99
Keywords: Probability, representativeness, misconceptions, high school students

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New exponential stability criteria for certain neutral differential equations with interval discrete and distributed time-varying delays

MELEK GÖZEN[1], CEMİL TUNÇ[2]

[1,2] Van Yuzuncu Yil University, Van, Turkey

emails: [1]melekgozen2013@gmail.com; [2]cemtunc@yahoo.com

In this paper, we give new sufficient conditions for the exponential stability of solutions to certain neutral dierential equations with discrete and distributed time-varying delays. Based on the some new definitions of a class of Lyapunov-Krasovskii functionals, a model transformation, the decomposition technique of constant coecients, the Leibniz-Newton formula and usage of a zero equation, some new delay-range-dependent exponential stability criteria are derived in terms of the linear matrix inequality (LMI) for the equations considered. We give an example to illustrate the eectiveness and improvement of the. results given.

MSC 2010: 34K20, 93D09, 93D20.
Keywords: Neutral differential equation, time-varying delay, exponentially stable.

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On the kink type and singular solitons solutions to the nonlinear partial differential equation

EDA GUNAYDIN[1], YUSUF GUREFE[2], TOLGA AKTURK[3]

[1,3] Ordu University, Ordu, Turkey
[2] Usak University, Usak, Turkey

emails: [1]eda-gunaydin@windowslive.com; [2]yusuf.gurefe@usak.edu.tr; [3]tolgaakturk@odu.edu.tr

In this study, the solutions of the Kadomtsev-Petviashvili equation were obtained by using the modified expansion function method. With this method, two and three dimensional graphics are drawn by selecting appropriate parameter values. It can be seen that the shapes of the obtained graphs correspond to the kink type and other soliton solutions graph. All the obtained solutions were checked with the help of the Wolfram Mathematica software, which provided the KP equation.

Following form the KP equation,

$$(u_t + 6uu_x + u_{xxx})_x - 3u_{yy} = 0, \tag{1}$$

MSC 2010: 35C07, 35C08, 35J60
Keywords: The modified expansion function method (MEFM), Kadomtsev-Petviashvili (KP) equation, the kink type soliton solution

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Lorentzian homogeneous generalized Ricci solitons of dimension $n \geq 3$

SİNEM GÜLER[1,2]

[1]Istanbul Technical University, Istanbul, Turkey
[2]Istanbul Sabahattin Zaim University, Istanbul, Turkey

emails: [1]singuler@itu.edu.tr; [2]sinem.guler@izu.edu.tr

In this study, we investigate the existence of Lorentzian homogeneous (generalized) Ricci solitons of dimension $n = 3$ and $n = 4$. It is known that under some conditions, three dimensional locally homogeneous Lorentzian manifolds are locally symmetric. Moreover, three dimensional locally symmetric Lorentzian manifolds which are not of constant sectional curvature are Walker manifolds, if they are not locally isometric to a Lorentzian product of a real line and a surface of constant Gauss curvature, [1, 2]. Motivated by these results, we construct several non-trivial examples of (generalized) Ricci solitons endowed with the three and four dimensional Walker metrics, [3].

MSC 2010: 53C21, 53C50, 53C25
Keywords: Ricci soliton, generalized Ricci soliton, Walker manifold
Acknowledgement: This work is supported by GAP project TGA-2018-41211 of Istanbul Technical University.

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Multi-party key exchange protocol and man in the middle attack

TUBA GÜLEŞCE TATLI[1], TURGUT HANOYMAK[2], ÖMER KÜSMÜŞ[3]

[1,2,3]Van Yüzüncü Yıl University, Van, Turkey

emails: [1]tubagulescetatli@gmail.com; [2]turguthanoymak@gmail.com; [3]omerkusmus@yyu.edu.tr

Key exchange protocols are used to generate a shared secret key between parties who want to communicate each other securely over an insecure channel. In this study, we first briefly mention about Diffie-Hellman protocol, then we generalize this to a multi-party type key exchange protocol and finally we give how an adversary can attack to this system by using the method of man in the middle attack, illustrating some concrete examples.

MSC 2010: 94A60, 94A62, 68P25
Keywords: Diffie-Hellman key exchange protocol, man in the middle attack, public key cryptography

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Simulation studies for credibility-based multi-objective programming problems with fuzzy parameters

HANDE GÜNAY AKDEMİR

Giresun University, Giresun, Türkiye

email: hande.akdemir@giresun.edu.tr

In this study, we discuss optimal decisions for hybrid models combining fuzzy chance-constraints and expected values of objective functions. Triangular, trapezoidal and non-linear fuzzy numbers are considered in problem parameters like demands and costs. Fortunately, the concept of fuzzy variables allows us to find risk-neutral decisions via expected values based on credibility measures. Finally, numerical simulations are presented to illustrate the efficiency.

MSC 2010: 90C29, 90C70, 03E72
Keywords: Multiple objective programming, fuzzy parameters, credibility measure, chance constraints, simulation

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Regularization of inverse coefficient determination problem in a hyperbolic problem

HAKKI GÜNGÖR

Ufuk University, Ankara, Turkey

email: hakki.gungor@ufuk.edu.tr

In this study, an inverse tension determination problem in a wave equation is investigated. Due to ill-posedness, a regularization process is carried out. After proving the existence and uniqueness of the solution, characterization of the solution is presented. The outcomes have also been tested with numerical examples.

MSC 2010: 65M32, 35L05
Keywords: Inverse Problem, adjoint method, wave equation

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Approximation by summation-integral type operators involving Brenke polynomials

ŞULE YÜKSEL GÜNGÖR

Gazi University, Ankara, Turkey

email: sulegungor@gazi.edu.tr

In this study, we introduce a sequence of summation-integral type operators linking generalized Brenke-Szász type and general Szász basis functions. A local and direct approximation theorem by means of Ditzian-Totik modulus of smoothness are obtained.The rate of convergence in terms of the Lipschitz class and the Lipschitz type maximal function is investigated.

MSC 2010: 41A25, 41A35, 41A36
Keywords: Brenke polynomials, Szász operators, Lipschitz class, Lipschitz type maximal function, degree of approximation

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Some notes on the order-to-topology continuous operators

KAZEM HAGHNEJAD AZAR

University of Mohaghegh Ardabili, Ardabil, Iran

email: haghnejad@uma.ac.ir

Let E be a Riesz space and F be a vector topology with topology τ. An operator T from E into F is said to be order-to-topology continuous whenever $x_\alpha \xrightarrow{o} 0$ implies $Tx_\alpha \xrightarrow{\tau} 0$ for each $(x_\alpha)_\alpha \subset E$. For each sequence $(x_n) \subset E$, if $x_n \xrightarrow{o} 0$ implies $Tx_n \xrightarrow{\tau} 0$, then T is called σ-order-to-topology continuous operator. The collection of all order-to-topology continuous operators will be denoted by $L_{o\tau}(E,F)$; the subscript $o\tau$ is justified by the fact that the order-to-topology continuous operators, that is,

$$L_{o\tau}(E,F) = \{T \in L(E,F) : \ T \text{ is order-to-topology continuous } \}.$$

Similarly, $L^\sigma_{o\tau}(E,F)$ will be denote the collection of all σ-order-to-topology continuous operators, that is,

$$L^\sigma_{o\tau}(E,F) = \{T \in L(E,F) : \ T \text{ is } \sigma - \text{order-to-topology continuous } \}.$$

For a normed space F, we write $L_{on}(E,F)$ and $L_{ow}(E,F)$ for collection of order-to-norm topology continuous operators and order-to-weak topology continuous operators, respectively. $L^\sigma_{on}(E,F)$ and $L^\sigma_{ow}(E,F)$ have similar definitions. Let E be a σ-Dedekind complete Riesz space and F be a normed Riesz space. If T is interval-bounded, then $T \in L^\sigma_{on}(E,F)$ if and only if T is order weakly compact. Let E be a Riesz space and let F a normed Riesz space with order unit. Then $L_{on}(E,F)$ is a band in $L_b(E,F)$. In this paper, we will study some properties of this new classification of operators. We will investigate the relationships between order-to-topology continuous operators with order continuous, order weakly compact and b-weakly compact operators, see [2, 3]

MSC 2010: 46B42, 47B60
Keywords: Riesz space, order-to-topology continuous, b-weakly compact operator.

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On mathematical aspects of blockchain architecture

TURGUT HANOYMAK[1], ATİLLA BEKTAŞ[2]

[1]Van Yüzüncü Yıl University, Van, Turkey
[2]Middle East Technical University, Ankara, Turkey

emails: [1]turguthanoymak@gmail.com; [2]bektasatilla@gmail.com

Blockchain technology is a distributed database that enables transfer of assets we value beyond the transfer of data made in many places such as multimedia, communication, web interface in today's internet world. In its broadest terms, blockchain is the delivery of central trust over the Internet, allowing the removal of a central server or trusted authority. In this study, we firstly give an overview of blockchain architecture and then present the mathematics behind blockchain such as finite fields, digital signature algorithms, hashing algorithms, public key cryptography, and so on. Furthermore, technical challenges and recent advances are briefly listed. We also summarize possible future trends for blockchain.

MSC 2010: 94A60, 94A62, 68P25
Keywords: Blockchain, bitcoin, digital signature, public key cryptography

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A new class of set-valued contractions and related results

HÜSEYİN IŞIK

Muş Alparslan University, Muş, Turkey

email: isikhuseyin76@gmail.com

The aim of this study to investigate the existence of solutions for nonlocal integral boundary value problem of Caputo type fractional differential inclusions. To achieve this goal, we take advantage of fixed point theorems for multivalued mappings satisfying a new class of contractive conditions in the setting of complete metric spaces. We derive new fixed point results which extend and improve the results in [1, 2, 3] and others by means of this new class of contractions. We also supply some examples to support the new theory.

MSC 2010: 34A08, 34A60, 47H10
Keywords: Multivalued maps, fixed points, fractional differential inclusions, nonlocal boundary conditions

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On numerical solution of an optimal control problem involving hyperbolic equation

SEDA İĞRET ARAZ

Siirt University, Siirt, Turkey

email: sedaaraz@siirt.edu.tr

In this paper, we presents a numerical algorithm for solving a class of optimal control problems with hyperbolic equation. We show that the optimal solution is exist and unique in a regular space. After obtaining adjoint problem and calculating derivative of the cost functional, numerical approximations are obtained via Gradient Method. Computational results demonstrate that the proposed method is able to generate good numerical approximations for optimal control problems.

MSC 2010: 49J20, 35L20, 49J50
Keywords: Optimal control, hyperbolic equations, Frechet differentiability

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Conditional expectation operators on measurable function spaces

MOHAMMAD REZA JABBARZADEH

University of Tabriz, Tabriz, Iran

email: mjabbar@tabrizu.ac.ir

In this note, we discuss matrix theoretic characterizations for weighted conditional type operators in some operator classes on $L^2(\Sigma)$ such as, self-adjoint, normal, quasinormal and positive operator classes. Also, we prove some basic results on the Moore-Penrose inverse and the Aluthge transformation of these type operators.

MSC 2010: 47G30 , 47B20, 47B38
Keywords: Conditional expectation, Moore-Penrose inverse, Aluthge transformation

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Inverse kinematics computation for a 6-DOF articulated robot arm using conformal geometric algebra

BAHAR KALKAN[1], ŞENAY BAYDAŞ[2], BÜLENT KARAKAŞ[3]

[1,2,3]Van Yuzuncu Yil University, Van, Turkey

emails: [1]baharkalkan1@gmail.com; [2]senay.baydas@gmail.com; [3]bulentkarakas@gmail.com

Geometric algebra provides a powerful computational framework for geometric applications in many areas including robotics. In fact, geometric algebra enables us to express fundamental robotics physics in a language that is free from coordinates or indices. The geometric algebra framework gives many equations a degree of clarity that is definitively lost in matrix algebra or tensor algebra. Geometric algebra represents orthogonal transformations more efficiently than the orthogonal matrices by reducing the number of coefficients. The rotation property can be applied to all objects in geometric algebra while it can be applied only on vectors in the quaternion algebra. Conformal geometric algebra is geometric algebra expanded to five dimensions from three dimensions. In conformal geometric algebra, in addition to the three-dimensional space, two more basic vectors are added, representing the location of the origin and the infinity. $\mathbb{R}^n$ is extended with unit vectors e_+ and e_- and $\mathbb{G}^{n+1,1}$ is constructed by defining $e_+^2 = 1$ and $e_-^2 = -1$. Define the origin $o = \frac{e_- - e_+}{2}$ and infinity $\infty = e_- + e_+$. These are null vectors:

$$o^2 = \infty^2 = 0.$$

In this algebra straight lines, planes, circles and spheres can be defined as vectors and rotations and translations can be defined by the rotor. These rotors can be applied to any object. In this paper, a new algorithm for the forward displacement analysis of an Articulated robot arm with six degrees of freedom mechanism based on geometric algebra (GA) will be presented. This paper describes a novel method for solving the inverse kinematics of this robot arm using conformal geometric algebra and proposes a geometric algebra (GA) based approach to carry out kinematics of given mechanism.

MSC 2010: 15A66, 53A17
Keywords: Geometric algebra (GA), inverse kinematics, robotics, conformal geometric algebra (CGA), Clifford algebra

On some vector valued multiplier spaces obtained by Zweir matrix method

RAMAZAN KAMA

Siirt University, Siirt, Turkey

emails: ramazankama@siirt.edu.tr

In this study, by using the Zweir matrix and a sequence of continuous linear operators, we introduce some vector valued multiplier spaces and summing operators associate with this spaces, respectively and study a series of some properties of them.

MSC 2010: 46B15, 40A05, 46B45
Keywords: Vector valued multiplier space, Zweir matrix, Summing operator

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$\Lambda-$ matrix as a summability operator and completeness of certain normed spaces via weakly unconditionally Cauchy series

MAHMUT KARAKUŞ[1], TUNAY BİLGİN[2]

[1,2] Van Yuzuncu Yil University, Van, Turkey

emails: [1]mkarakus@yyu.edu.tr; [2]tbilgin@yyu.edu.tr

Let $\sum_i x_i$ be a series in a real normed space. Then the series is said to be weakly unconditionally Cauchy if the sequence $\left(\left(\sum_{i=1}^{n} x_{\pi(i)}\right)_n\right)$ is weakly Cauchy sequence, for every permutation π of $\mathbb{N}$, the set of positive integers X [1]. Some characterizations of weakly unconditionally Cauchy series are well-known facts. For example, the series $\sum_n x_n$ in a Banach space is weakly unconditionally Cauchy iff it is a c_0- multiplier convergent series [2].

In this study we interest some new characterizations of weakly unconditionally Cauchy series by completeness of certain normed spaces which are obtained from summability matrix $\Lambda = (\lambda_{nk})$ of Móricz [3] defined by

$$\lambda_{nk} = \begin{cases} \frac{\lambda_k - \lambda_{k-1}}{\lambda_n} & , \quad (1 \leq k \leq n) \\ 0 & , \quad (k > n) \end{cases} . \tag{1}$$

Here, $\lambda = (\lambda_k)$ is a strictly increasing sequence of positive reals tending to infinity, i.e., $0 < \lambda_1 < \lambda_2 < ...$ and $\lim_k \lambda_k = \infty$.

MSC 2010: 46B15, 46A45, 40A05
Keywords: Weakly unconditionally Cauchy series, matrix summability, completeness of normed spaces

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The maximal function in Sobolev spaces

YASİN KAYA

Dicle University, Diyarbakır, Turkey

email: ykaya@dicle.edu.tr

The theory of Hardy-Littlewood maximal function and Sobolev spaces one of the most important topic in analysis

In this presentation, I give an overview of the development of the maximal function in Sobolev spaces and show a result in this frame.

MSC 2010: 46E35, 47G10, 26A42
Keywords: Maximal function, Sobolev spaces, sublinearity

A novel Lyapunov type inequality for quasilinear impulsive systems

ZEYNEP KAYAR

Van Yuzuncu Yil University, Van, Turkey

email: zeynepkayar@yyu.edu.tr

We establish Lyapunov-type inequality and for Dirichlet problem associated with the quasilinear impulsive system involving the (p, q)-Laplacian operator. This inequality is used to obtain disconjugacy criterion and to find lower bounds for eigenvalues associated to related eigenvalue problems. Our results not only improve the recent related results and that of [1, 2] but also generalize them to the impulsive case.

MSC 2010: 34A37, 34C10, 34A34
Keywords: Lyapunov type inequality, impulse, quasilinear systems

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Total dominator coloring of a graph

ADEL P. KAZEMI

University of Mohaghegh Ardabili, Ardabil, Iran

email: adelpkazemi@yahoo.com

A total dominator coloring of a graph G is a proper coloring of G in which each vertex of the graph is adjacent to every vertex of some color class. The total dominator chromatic number of a graph is the minimum number of color classes in a total dominator coloring of it. Here, we talk on the total dominator coloring of a graph by giving some tight bounds for the total dominator chromatic number of a graph, a tree, join of two graphs and Nordhaus-Gaddum-like relations.

MSC 2010: 05C15, 05C69
Keywords: Total dominator coloring, Total dominator chromatic number, total domination number, central graph, Nordhaus-Gaddum relation

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Invariant submanifolds of statistical Kenmotsu manifolds and their curvatures

MOHAMMAD BAGHER KAZEMI BALGESHIR

University of Zanjan, Zanjan, Iran

email: mbkazemi@znu.ac.ir

A Riemannian manifold $(\bar{M}, g)$ with an affine and torsion free connection $\bar{\nabla}$ satisfying

$$\bar{\nabla}_V g(U,W) = \bar{\nabla}_U g(V,W) \quad \forall U,V,W \in \mathcal{T}(\bar{M}), \tag{1}$$

is called a statistical manifold [1, 2]. Moreover, there is an affine connection $\bar{\nabla}^*$ on $\bar{M}$ which is called the dual connection of $\bar{\nabla}$ with respect to the g, such that

$$Ug(V,W) = g(\bar{\nabla}_U V, W) + g(V, \bar{\nabla}^*_U W). \tag{2}$$

In this paper, we study statistical manifolds which admit an almost contact and Kenmotsu structure [3]. We investigate the shape operator of invariant submanifolds of statistical Kenmotsu manifolds and prove it vanishes on these submanifolds if the the structure vector field is tangent to the submanifold.

MSC 2010: 53C15, 53C40, 60D05
Keywords: Kenmostu manifold, statistical structure, invariant submanifold

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Reduction of Navier-Stokes equation to a linear equation

WALEED S. KHEDR

Cairo, Egypt

email: waleedshawki@yahoo.com

In this article, we provide two theorems on pointwise coincidence between solutions of Navier-Stokes equation and solutions of standard linear second order parabolic equations with the same data. We show that the convection, the pressure, and the external forces (if applied) are governed by some sort of balance independent of the equation that governs the solution itself. In light of the well establishment of the theory of existence, regularity and uniqueness of linear second order parabolic equations, this result represents an important step to fully understand the qualitative properties of the solutions to Navier-Stokes equation. **Details:** The initial profile is $\mathbf{v}_0$ and the boundary conditions (in case of bounded domains) are denoted by $\mathbf{v}^*$. The model equation in hand is:

$$\begin{cases} \mathbf{v}_t + (\mathbf{v}\cdot\nabla)\mathbf{v} - \mu\Delta\mathbf{v} = -\nabla p + \mathbf{f}, \quad \nabla\cdot\mathbf{v} = 0 \quad \text{in} \quad \Omega_t, \\ \mathbf{v}(\mathbf{x},0) = \mathbf{v}_0(\mathbf{x}) \quad \text{in} \quad \Omega_0, \\ \mathbf{v}(\mathbf{x},t) = \mathbf{v}^*(\mathbf{x}_{n-1},t) \quad \text{on} \quad \partial\Omega_t, \end{cases} \tag{1}$$

where Ω_t and $\partial\Omega_t$ denotes the fixed boundary if the domain is bounded. The solution $\mathbf{v}$ is the vector field representing the velocity of the flow in each direction, and its rotation $\omega = \nabla\times\mathbf{v}$ is the vorticity. Note that $\nabla\cdot\omega = 0$ in $\overline{\Omega}_t$ by compatibility. The solution is investigated in light of the most common classical definition of weak solutions in order to ensure the possibility of generalizing the subsequent results to the widest classes of possible solutions. In particular, we follow the definition of weak solutions introduced first by Leray [1, 2, 3]. Further, the investigation in this article establishes rigorously the previously investigated results in [4, 5].

MSC 2010: 76D03, 76D05, 76M30, 76R10
Keywords: Fluid Mechanics, Navier-Stokes equation, convection
Acknowledgement: This is an extended abstract of the article submitted to arXiv on 24th of January 2018 with submission number arXiv:submit/2142016.

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A note on the annihilator of certain local cohomology modules

AHMAD KHOJALI

University of Mohaghegh Ardabili, Ardabil, Iran

email: Khojali@uma.ac.ir

Let R be a commutative Noetherian ring of dimension d and $\mathfrak{a}$ an ideal of R. It is easily seen that $\frac{R}{\mathrm{Ann}_R(H_{\mathfrak{a}}^{d-1}(R))} \hookrightarrow \mathrm{Hom}_R(H_{\mathfrak{a}}^{d-1}(R), H_{\mathfrak{a}}^{d-1}(R))$. Vanishing of $\mathrm{Ann}_R(H^{d-1}$ is interesting. In this direction, we consider the question whether non-vanishing of $H_{\mathfrak{a}}^{d-1}(R)$ is equivalent to the vanishing of its annihilator. In the case $(R, \mathfrak{m})$ is a regular local ring containing a field and $H_{\mathfrak{a}}^{i}(R) \neq 0$ for a given integer i, then in characteristic zero Lyubeznik [5] and in characteristic $p > 0$ Huneke and Koh [3] showed that $\mathrm{Ann}_R H_{\mathfrak{a}}^{i}(R) = 0$. Here are some attempts to compute $\mathrm{Ann}_R H_{\mathfrak{a}}^{i}(R)$. (1) If R is a local complete ring R with $H_{\mathfrak{a}}^{i}(R) = 0$ for every $i \neq \mathrm{ht}(\mathfrak{a})$, then $\mathrm{Ann}_R(H_{\mathfrak{a}}^{\mathrm{ht}(\mathfrak{a})}(R)) = 0$ (see [2]). (2) If R is a complete Gorenstein local domain with some mild assumptions $\mathrm{Ann}_R(H_I^{i}(R)) = 0$, where $i = \mathrm{grade}(\mathfrak{a}, R)$ (see [6]). (3) If R is complete and local, then $\mathrm{Ann}_R H_{\mathfrak{a}}^{d}(R) = \cap \mathfrak{q}$, that $\mathfrak{q}$ are primary components of (0) with $\dim \frac{R}{\mathfrak{q}} = \dim R$ and $\mathrm{rad}(\mathfrak{a} + \mathfrak{q}) = \mathfrak{m}$ (see [4]).

In this note we consider the annihilator of some certain local cohomology modules and some vanishing results of these modules will be considered. In particular, it is proved that: (1) If $(R, \mathfrak{m})$ be a local domain such that $H_{\mathfrak{a}}^{d}(R) = 0$ and $H_{\mathfrak{a}}^{d-1}(R)$ is not Artinian, then $\mathrm{Ann}_R H_{\mathfrak{a}}^{d-1}(R) = 0$. (2) Let $(R, \mathfrak{m})$ be a Cohen-Macaulay local ring and $\mathfrak{a}$ a 1-dimensional ideal. Then, $H_{\mathfrak{a}}^{d-1}(R)$ is not Artinian.

MSC 2010: 13D45
Keywords: Annihilator of local cohomology, non-Artinian local cohomology, Buchsbaum type modules
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Sufficient conditions for global asymptotic stability of neural networks with time-varying delays

ERDAL KORKMAZ[1], CEMİL TUNC[2]

[1] Mus Alparslan University, Mus, Turkey
[2]Van Yuzuncu Yil University, Van, Turkey

emails: [1]korkmazerdal36@hotmail.com ; [2]cemtunc@yahoo.com;

In this paper, for global asymptotic stability of the equilibrium point of neural networks with delays is obtained some new sufficient conditions by using Lyapunov technique. The results obtained have shown to improve the previous results derived in the literature . The results are supported by a few examples.

MSC 2010: 34B18, 34B25
Keywords: Lyapunov functionals, global asymptotic stability, delay differential equations.

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On the cubic nonlinear Shrodinger's equation with repulsive delta potential

ZELİHA KÖRPINAR[1], FATİH COŞKUN[2]

[1,2]Muş Alparslan University, Muş, Turkey

emails: [1]zelihakorpinar@gmail.com; [2]fatihcoskun323@gmail.com

In this work, is introduced to obtain approximate solutions of the cubic nonlinear Shrödinger's equation (NLSE) with repulsive delta potential subject to certain initial conditions by using Residual power series method (RPSM). The consequens show that this method is efficient and convenient and can be applied to a large sort of problems. The approximate solutions are compared with the known exact solutions.

MSC 2010: 35L05, 58Z05
Keywords: Residual power series method, cubic nonlinear Shrödinger's equation, potential

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On numerical solutions for fractional (1+1)-dimensional Biswas-Milovic equation

ZELİHA KÖRPINAR[1], MUSTAFA İNÇ[2]

[1]Muş Alparslan University, Muş, Turkey
[2]Fırat University, Elazığ, Turkey

emails: [1]zelihakorpinar@gmail.com; [2]minc@firat.edu.tr

In this work, numerical solutions are obtained for fractional (1+1)-dimensional Biswas-Milovic equation that defines the long-space optical communications by using the residual power series method (RPSM). The RPSM gets Maclaurin expansion of the solution. The solutions of present equation are computed in the shape of quickly convergent series with quickly calculable fundamentals by using mathematica software package. Explanation of the method is given graphical consequens and series solutions are made use of to represent our solution. The found consequens show that technique is a power and efficient method in conviction of solution for the fractional (1+1)-dimensional Biswas-Milovic equation.

MSC 2000: 35L05, 58Z05
Keywords: Residual power series method, (1+1)-dimensional Biswas-Milovic equation, Series solution.

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On inextensible flow with Schrödinger flow

ZELİHA KÖRPINAR[1], TALAT KÖRPINAR[2], SELÇUK BAŞ[3], M. TALAT SARIAYDIN[4]

[1,2,3]Muş Alparslan University, Muş, Turkey
[4]Selçuk University, Konya, Turkey

emails: [1]zelihakorpinar@gmail.com; [2]talatkorpinar@gmail.com;[3]selcukbas79@gmail.com ; [4]talatsariaydin@gmail.com

In this work, we study inextensible flows with differential geometry properties of surfaces by using Bäcklund transformations of integrable geometric Schrödinger flow. We give some new solutions by using the Schrödinger flow. Moreover, we characterize some solutions of curvature and torsion.

MSC 2000: 53C41, 53A10
Keywords: Schrödinger flow, $\mathbb{E}^3$, extended Riccati mapping method, Bäcklund transformations.

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Idempotent unit group in commutative group rings of direct products

OMER KUSMUS[1], I. HAKKI DENIZLER[2], NECAT GORENTAS[3]

[1,2,3] Van Yuzuncu Yil University, Van, Turkey

emails: [1]kusmuso@gmail.com; [2]ihdenizler@gmail.com; [3]negorentas@yyu.edu.tr

Let $\mathcal{U}(RG)$ denotes the unit group of the group ring RG of a given group G over the ring R and also $\mathcal{V}(RG)$ shows the normalized unit subgroup in $\mathcal{U}(RG)$. Idempotent units in $\mathcal{V}(RG)$ is formally defined as

$$< \sum_{g \in G} e_g g : \forall g \in G, e_g = e(g) \in R, e_g^2 = e_g, \sum_{g \in G} e_g = 1, e_g e_h \stackrel{g \neq h}{=} 0 >$$

and displayed by $Id(RG)$. In this study, since R is a commutative unitary ring, both G and H are abelian groups, we search for some necessary and sufficient conditions for

$$Id(R(G \times H)) = Id(RG) \times Id(RH)$$

MSC 2010: 16S34, 16U60, 20K10, 20K20, 20K21
Keywords: Idempotents, nilpotents, units, unit group, group rings

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Investigating a quadratic Bezier curve according to N-Bishop frame

HATİCE KUŞAK SAMANCI[1], MUHSİN İNCESU[2]

[1] Bitlis Eren University, Bitlis, Turkey
[2] Muş Alparslan University, Muş, Turkey

emails: [1]hkusak@beu.edu.tr; [2]m.incesu@alparslan.edu.tr

It is known that Bezier curve is one of the effective method for computer aided geometric design (CAGD). The second degree of general Bezier curve is called as a quadratic Bezier curve. Also, N-Bishop frame is a new Bishop frame introduced by O. Keskin and Y. Yaylı in [3]. In our work, a brief summary about a quadratic Bezier curve are firstly presented. Moreover, we research some geometric properties of the quadratic Bezier curve according to N-Bishop frame. Finally, we obtained the N-Bishop curvatures and derivative formulas for this curve.

MSC 2010: 53A04, 68U07
Keywords: Bezier Curves, CAGD, N-Bishop, curvature

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Review on fuzzy thermal image processing applications

FATİH KUTLU[1], ÖZKAN ATAN[2]

[1,2]Van Yüzüncü Yıl University, Van, Turkey
emails: [1]fatihkutlu@yyu.edu.tr; [2]oatan@yyu.edu.tr

The main purpose of this study is to improve the edge detection performance in thermal images with a new fuzzification method. In this method, contrary to other studies in the literature, the thermal image is fuzzificated separately from the RGB channels and edge detection performance is improved with the aid of aggregation operators.

MSC 2010: 90C70, 94D05
Keywords: thermal image processing, fuzzy sets, edge detection
Acknowledgement: This work was supported by Research Fund of the Van Yüzüncü Yıl University. Project Number: FBA-2018-6531

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On temporal intuitionistic fuzzy De Morgan triplets

FATİH KUTLU[1], FERİDE TUĞRUL[2], MEHMET ÇİTİL[3]

[1]Van Yüzüncü Yıl University, Van, Turkey
[2,3]Kahramanmaraş Sütçü İmam University, Kahramanmaraş, Turkey

emails: [1]fatihkutlu@yyu.edu.tr [2]feridetugrul@gmail.com; [3]citil@ksu.edu.tr

The aim of this study is to define negator, t-norm and t-conorms, which is the generalization of negation, conjunctions and disconjunctions in the temporal intuitionistic fuzzy sets and to examine the De Morgan relations between these concepts.

MSC 2010: 94D05, 03E72
Keywords: De Morgan triplet, t-norm, t-conorm, negation, temporal intuitionistic fuzzy sets.

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An interplay between Riemann integrability and weaker forms of continuity

NISAR A. LONE

University of Kashmir, Srinagar, India

email: nisarsultan@gmail.com

It is folklore that a continuous function $f : [0, 1] \longleftrightarrow \mathbb{R}$ is Riemann integrable. The result holds true if the real space R is replaced by a Banach space X. The result doesn't hold true if the function is assumed to be continuous with respect to a topology weaker than the norm topology. In this paper characterization of Banach spaces will be discussed in terms of the Riemann integrability of functions which are continuous in weaker topologies. The reverse part of the problem which results in introduction of the property of Lebesgue will also be discussed.

MSC 2010: 46G10, 46G12
Keywords: Riemann integral, weak continuity, Grothendieck space

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Ehrlich-Aberth's type method with King's correction for the simultaneous approximation of polynomial zeros

ROSELAINE NEVES MACHADO[1], LUIZ GUERREIRO LOPES[2]

[1]Federal Institute of Rio Grande do Sul, Bento Gonçalves, RS, Brazil
[2]Faculty of Exact Sciences and Engineering, University of Madeira, Funchal, Madeira Is., Portugal

emails: [1]roselaine.machado@bento.ifrs.edu.br; [2]lopes@uma.pt

There are many simultaneous iterative methods for approximating complex polynomial zeros, from more traditional numerical algorithms, such as the well-known third order Ehrlich-Aberth's method [1, 2], to the more recent ones. In this paper, we present a new combined method for the simultaneous determination of complex zeros of a polynomial, which uses the Ehrlich-Aberth iteration and a correction based on King's method [3]. Using King's correction, the order of convergence of the basic method is increased from 3 to 6. Numerical examples are given to illustrate the accuracy and computational efficiency of the proposed combined method for the simultaneous approximation of polynomial zeros.

MSC 2010: 30C10, 65H04, 65Y20
Keywords: Polynomial zeros, simultaneous iterative methods, combined iterative methods
Acknowledgement: This work was supported by the Federal Institute of Rio Grande do Sul (IFRS), Brazil.

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Direct sum of neighborhoods in locally convex cones

MOHAMMAD REZA MOTALLEBI

University of Mohaghegh Ardabili, Ardabil, Iran

email: motallebi@uma.ac.ir

Using the direct sum of neighborhoods, we define the product cone topologies in locally convex cones. The polar of every product neighborhood may be written as the direct sum of its components which leads us to investigate the duality properties of product cones. In particular, we conclude that the weak topology on product cone is the locally convex product cone of its components.

MSC 2010: 20K25, 46A20, 46A03.
Keywords: Product and direct sum, duality, locally convex cone.

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A note on superposition operators in Fibonacci sequence spaces $l_p(F)$

OĞUZ OĞUR

Giresun University, Giresun, Turkey

email: oguz.ogur@giresun.edu

Let X and Y be sequence spaces. A superposition operator P_g on X is a mapping from X to Y defined by $P_g(x) = (g(k, x_k))_{k=1}^{\infty}$, where $g : \mathbb{N} \times \mathbb{R} \to \mathbb{R}$ with $g(k, 0) = 0$ for all $k \in \mathbb{N}$. In this paper, we study on characterization of superposition operator P_g acts from $c_0(F)$ and $\ell_p(F)$ to $\ell_q(F)$, $1 \leq p, q \leq \infty$, where $F = (f_{nk})_{n,k=1}^{\infty}$ is Fibonacci matrix such as: $f_{nk} = \frac{f_k}{f_{n+2}-1}$ if $1 \leq k \leq n$ and $f_{nk} = 0$ if $k > n$. Also, we give the necessary and sufficient conditions for continuity of $P_g : \ell_p(F) \to \ell_q(F)$.

MSC 2010: 47H30, 46A45, 40A05, 46B45
Keywords: Superposition operator, Fibonacci sequence space, matrix transformation

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On contact surgeries and a counterexample

SİNEM ONARAN

Hacettepe University, Ankara, Turkey

email: sonaran@hacettepe.edu.tr

Contact surgeries have long been an essential tool in the study of contact 3-manifolds. Contact surgeries are roughly defined as removing a neighborhood of a Legendrian knot and gluing a contact solid torus back for which we can extend the contact structure on its boundary to the inside. In this talk, we will focus on the behaviour of contact structures under contact $(+n)$–surgeries along Legendrian knots where the surgery slope is measured with respect to the contact framing of the Legendrian knot. We give a counterexample to a conjecture by James Conway on overtwistedness of manifolds obtained by contact $(+n)$–surgery.

MSC 2010: 53D10, 57M25, 57R65,
Keywords: Contact structure, contact surgery, Legendrian knot
Acknowledgement: This work is supported by the Scientific and Technological Research Council of Turkey, T'UBİTAK Project No:115F519.

Singular eigenvalue problems via Hilfer derivative

RAMAZAN OZARSLAN[1], ERDAL BAS[2], AHU ERCAN[3]

[1,2,3]Firat University, Elazig, Turkey

emails: [1]ozarslanramazan@gmail.com; [2]erdalmat@yahoo.com; [3]ahuduman24@gmail.com

In this article singular eigenvalue problem is considered with Hilfer derivative. Self-adjointness of the operator is analyzed and some spectral properties are given. Let $\alpha \in (0,1)$ and $\beta \in [0,1]$. Singular eigenvalue problem with Hilfer derivative is defined as follows

$$\mathcal{L}_{\alpha[C]} y_\lambda(x) + \lambda w_\alpha(x) y_\lambda(x) = 0 \tag{1}$$

where $p(x) \neq 0, w_\alpha(x) > 0 \ \forall x \in (0,\pi], w_\alpha(x)$ is weight function and p, q are real valued continuous functions in interval $(0,\pi]$ and $\frac{y_\lambda(x)}{x} \in C[0,\pi]$. The boundary conditions for the problem (1-3) are the following:

$$c_1 I_{0+}^{(1-\alpha)(1-\beta)} p(0) D_{\pi-}^{\alpha,\beta} y(0) + c_2 I_{\pi-}^{\beta(1-\alpha)} y(0) = 0, \tag{2}$$

$$d_1 I_{0+}^{(1-\alpha)(1-\beta)} p(\pi) D_{\pi-}^{\alpha,\beta} y(\pi) + d_2 I_{\pi-}^{\beta(1-\alpha)} y(\pi) = 0, \tag{3}$$

where $c_1^2 + c_2^2 \neq 0$ and $d_1^2 + d_2^2 \neq 0$.

MSC 2010: 26A33, 34A08, 35P05
Keywords: Fractional, Hilfer, singular, spectral.

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An efficient TVD-WAF scheme application for the 2D shallow water equations on unstructured meshes

NURAY ÖKTEM

Ankara Yıldırım Beyazıt University, Turkey

email: nbozkaya@gmail.com

The solutions of 2D shallow water equations (SWE) are often used to study free surface flows such as shallow lakes, dam-breaks, inundations, wide rivers, estuaries and coastal zones. Although the flow is 3D in nature, the simplified 1D or 2D shallow flow simulations may still cover a considerable amount of real flow characteristics and provide practical and quick numerical solutions in many engineering applications.

In this study a weighted averaged flux (WAF) method application [1] is presented for the solution of SWE with a new total variation diminishing (TVD) flux-limiter tool on unstructured meshes. The numerical solver is based on a 2nd order accurate (in both time and space) cell-centered finite volume formulation and it is especially constructed on an unstructured triangular mesh to be used for complex flow geometries as well. For the interface flux computations WAF alone provides 2nd order accuracy spatially but brings some numerical oscillations besides. In order to smooth these oscillations and ensure the stabilization of the interface flux solutions WAF is coupled with a new version of TVD for unstructured meshes. Shock wave development and propagation at interfaces is another difficulty in the resolution of SWE numerical models due to their hyperbolic nature. Thus in addition to TVD theory, an HLLC(Harten, Lax, van Leer-Contact wave, [2]) Riemann solver is required and so is implemented to support the shock capturing property and contact discontinuities recognition property of the coupled TVD-WAF algorithm. Moreover, a two step Runge-Kutta algorithm is applied to keep the 2nd order accuracy in time direction. Finally, a novel software written in Fortran programming language [3] for this numerical process is utilized and efficient numerical results are obtained for various benchmark problems. As a further application an open-channel junction flow is considered.

MSC 2010: 35Q35, 35L65, 65M08, 74J40
Keywords: Unstructured mesh, WAF, shallow flow, HLLC Riemann solver

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Blow up of solutions for a quasilinear Kirchhoff-type wave equations with degenerate damping terms

ERHAN PİŞKİN[1], FATMA EKİNCİ[2]

[1,2]Dicle University, Diyarbakır, Turkey

emails: [1]episkin@dicle.edu.tr; [2]ekincifatma2017@gmail.com

In this work, we analyze the influence of degenerate damping terms and source terms on the solutions of the quasilinear Kirchhoff-type wave equations. We will show the blow up of solutions in finite time with positive initial energy. This improves earlier results in the literature ([1], [2]).

MSC 2010: 35B44, 35L53
Keywords: Blow up, Kirchhoff-type wave equations, Degenerate damping terms

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Semi-tensor bundle and the vertical lift of tensor fields

MURAT POLAT

Atatürk University, Erzurum, Turkey

email: murat_sel_22@hotmail.com

The present paper is devoted to some results concerning the vertical lift of tensor fields of type (p,q) from manifold M to its semi-tensor bundle tB of type (p,q).

MSC 2010: 53A45, 55R10, 57R25
Keywords: Vector field, pull-back bundle, semi-tensor bundle.

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New Lie group of transformation for the non-Newtonian fluid flow narrating differential equations

KHALIL UR REHMAN[1], M.Y. MALIK[2]

[1,2] Quaid-i-Azam University Islamabad, Pakistan

emails: [1]krehman@math.qau.edu.pk; [2]drmymalik@qau.edu.pk

In this endeavour, a new Lie point of transformation for the fluid flow narrating differential equations are proposed. For this purpose a non-Newtonian fluid named tangent hyperbolic fluid is considered towards the flat surface in a magnetized flow field in the presence of both the heat and mass transfer characteristics. In addition, equation of concentration admits the role of chemically reactive species. A mathematical model in terms of the coupled PDE's is constructed. Lie group of analysis is performed to yield the Lie point of transformation for the tangent hyperbolic fluid flow narrating differential equations when both the heat and mass transfer individualities are taken into account. The resultant system of PDE's is reduced into the system of ODE's via obtained set of transformation. A self-coded computational scheme is executed and outcomes in this regard are reported by way of both the graphical and tabular structures.

MSC 2010: 22E70, 57S15, 65Z05
Keywords: Lie group of transformation, non-Newtonian fluid model, shooting method

Reproducing kernel method with Bernstein polynomials for fractional boundary value problems

MEHMET GIYAS SAKAR[1], ONUR SALDIR[2], FEVZİ ERDOGAN[3]

[1,2,3] Van Yuzuncu Yil University, Van, Turkey

emails: [1]giyassakar@hotmail.com; [2]onursaldir@gmail.com; [3]fevzier@gmail.com

In this article, a novel approach is introduced for numerical solution of linear and nonlinear boundary value problems with fractional order. Fractional derivative are taken in Caputo sense. This approach is based on reproducing kernel with Bernstein polynomials. So as to show the effect of the method, results are given as graphically and in tabulated forms.

MSC 2010: 46E22, 65Z05, 26A33
Keywords: Caputo derivative, reproducing kernel, Bernstein polynomials, boundary value problem

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A numerical approach for time-fractional Kawahara equation with reproducing kernel method

ONUR SALDIR[1], MEHMET GIYAS SAKAR[2], FEVZİ ERDOGAN[3]

[1,2,3]Van Yuzuncu Yil University, Van, Turkey

emails: [1]onursaldir@gmail.com; [2]giyassakar@hotmail.com; [3]ferdogan@yyu.edu.tr

We introduced a new approach based on reproducing kernel method for time fractional Kawahara equation. Approximate solution and convergence analysis of method are given. To show the power and effect of the method, an example is solved and results are given as tables and graphics. The results show that the method very convenient and efficient for Kawahara equation.

MSC 2010: 35R11, 46E22
Keywords: Reproducing kernel method, Kawahara equation, Caputo derivative, convergence

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Rings without a middle class: past and recent

BÜLENT SARAÇ

Hacettepe University, Ankara, Turkey

email: bsarac@hacettepe.edu.tr

In 1964, Barbara Osofsky ([8]) proved her celebrated theorem which states that any ring R with identity is semisimple artinian if and only if every cyclic right (or left) R–module is injective. This happened to be the first step in search of characterizations of rings by homological properties of some certain type of their modules. As a part of this long standing research program, a new type of rings were introduced in 2011 (see [3]) and has been studied extensively by several authors including the speaker. In this talk, I would like to present some new ideas from torsion theory by which we can unify the developing theory of rings of the title and show how these ideas can be effective in exploring many significant properties of these rings. In the last part of my talk, I would also like to discuss the case, which is still a mystery, where the rings are not right noetherian and apply our new techniques to work out this mysterious case.

MSC 2010: 16D50, 16D70, 16P20
Keywords: Injective module, poor module, injectivity domain, QI-, PCI-, V-ring
Acknowledgement: This talk is mainly based on results obtained under the research project supported by TUBITAK with project number 117F084.

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Some applications about Mobius function

UMİT SARP[1], DAEYOUL KIM[2], SEBAHATTİN IKIKARDES[3]

[1,3]Balikesir University, Balikesir, Turkey
[2]Chon-buk National University, 567 Baekje-daero, Republic of Korea

emails: [1]umitsarp@ymail.com; [2]kdaeyeoul@jbnu.ac.kr; [3]skardes@balikesir.edu.tr

In this paper, according to some numerical computational evidence, we investigate and prove certain relations and properties on Möbius function and some related functions.

MSC 2010: 11M36, 11F11, 11F30
Keywords: Möbius function, divisor function, perfect number.
Acknowledgement: This work was supported by Balikesir University Research, Grant No: 2017/20.

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Estimation of parameters of Gumbel distribution data

SANEM SEHRIBANOGLU

Van Yuzuncu Yil University, Van, Turkey

email: sanem@yyu.edu.tr

The Gumbel distribution is one of the particular states of generalized extreme value distribution (GEV). Probabilistic extreme value theory is a interesting and fascinating a great variety of applications. In probability theory and statistics, this distribution is used to model for the extremes (maximum or minimum) observations. The Gumbel distribution are frequently applied to forecast of natural events such as floods, air pollution, extreme sea levels, hydrology, meteorology, climatology, insurance, finance, geology and seismology.

In this paper, The parameters are estimated using maximum likelihood and Bayesian estimation procedure. In additionally to this inferences are used The Newton-Raphson algorithm and Markov Chain Monte Carlo(MCMC) simulation method.

MSC 2010: : 62H10, 62F15
Keywords: Generalized extreme value distribution, Gumbel distribution, MLE, Bayesian estimation

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Some convergences in metric spaces

GUZİDE SENEL

Amasya University, Amasya, Turkey

email: g.senel@amasya.edu.tr

Porosity was defined by Denjoy in [1]. A detailed information about this type porosity was given by Thomson in [1]. The notion of porosity can also be used in metric space by replacing intervals with balls (see [3]) In this study, I will define a new type of convergence for metric valued sequences. Then I will give some properties of this new concept.

MSC 2010: 47A12, 15A60, 26C10,30C15
Keywords: Metric space, sequence, porosity

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Existence of positive solutions for boundary value problems of nonlinear fractional differential equations

TUGBA SENLIK CERDIK[1], FULYA YORUK DEREN[2], NUKET AYKUT HAMAL[3]

[1]Izmir, Turkey
[2,3]Ege University, Izmir, Turkey

emails: [1]tubasenlik@gmail.com; [2]fulya.yoruk@ege.edu.tr; [3]nuket.aykut@ege.edu.tr

This study deals with the existence result of positive solution for the fractional boundary value problem. The arguments are based upon a fixed point theorem in a cone. Its application is also given.

MSC 2010: 34B10, 34B15, 34B18.
Keywords: Boundary value problem, positive solution, fractional differential equation.

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Elliptic curves containing sequences of consecutive cubes

GÖKHAN SOYDAN[1], GAMZE SAVAŞ ÇELİK[2]

[1,2]Uludağ University, Bursa, Turkey

emails: [1]gsoydan@uludag.edu.tr; [2]gamzesavascelik@gmail.com

Let us consider a rational elliptic curve given by a Weierstrass equation

$$y^2 + a_1xy + a_3y = x^3 + a_2x^2 + a_4x + a_6 \tag{1}$$

with $a_1, \cdots, a_6 \in \mathbb{Q}$. We will say that the points $(x_i, y_i), i = 1, \cdots, k$ on the curve (1) are in arithmetic progression of length k if the sequence $x_1, x_2, \cdots, x_k$ forms an arithmetic progression (AP for short). Firstly, Lee and Vélez, [5], found infinitely many curves of type $y^2 = x^3 + a$ containing $k = 4$-length APs. Then many authors considered elliptic curves containing $k \geq 8$-length APs (see [1], [2], [6]).

Recently, Kamel and Sadek, [4], considered sequences of rational points on elliptic curves given by the equation

$$y^2 = ax^3 + bx + c \tag{2}$$

over $\mathbb{Q}$ whose $x-$coordinates form a sequence of "consecutive squares".

In this work, we are interested in sequences of rational points on elliptic curves with the equation (2) whose $x-$coordinates form a sequence of "consecutive cubes". We show that elliptic curves given by the equation (2) with $5-$term sequences of rational points whose $x-$coordinates are elements of a sequence of consecutive cubes in $\mathbb{Q}$ parametrized by an elliptic surface whose rank is positive. This implies the existence of infinitely many such elliptic curves. We also show that these five rational points in the sequence are linearly independent in the group of rational points of the elliptic curve they lie on. Especially, we introduce an infinite family of elliptic curves of rank≥ 5 [3].

MSC 2010: 14G05, 11B83.
Keywords: Elliptic curves, rational points, sequences of consecutive cubes.

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Jones polynomial for graphs of twist knots

ABDULGANİ ŞAHİN[1], BÜNYAMİN ŞAHİN[2]

[1]Ağrı İbrahim Çeçen University, Ağrı, Turkey
[2]Bayburt University, Bayburt, Turkey

emails: [1]rukassah@gmail.com; [2]bsahin@bayburt.edu.tr;

Graphs are have made a great contribution to the development of algebraic topology. Along with this support, knot theory has taken an important place in low dimensional manifold topology. In 1984, Jones introduced a new polynomial for knots. The discovery of that polynomial opened a new era in knot theory. In a short time, this polynomial was defined by algebraic arguments and its combinatorial definition was made. The Jones polynomials of knot graphs and their applications were introduced by Murasugi. T. Uur and A. Kopuzlu found an algorithm for the Jones polynomials of torus knots $K(2,q)$ in 2006. In this paper, we compute the Jones polynomials for graphs of twist knots. We will consider signed graphs associated with each twist knot diagrams.

MSC 2000: 57M15, 57M25, 57M27
Keywords: Twist knots, knot graph, Jones polynomial

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Kernel stable and uniquely generated modules

SERAP ŞAHİNKAYA[1], TRUONG CONG QUYNH[2]

[1]Gebze Technical University, Kocaeli, TURKEY
[2] Danang University, DaNang City, VIETNAM

emails: [1]ssahinkaya@gtu.edu.tr; [2] tcquynh@dce.udn.vn

Module theoretic notion of annihilator-stable rings which was defined recently by Nicholson [2] is defined and some characterizations of it studied. M is called kernel-stable module if every element $\alpha \in End(M)$ satisfies the following condition: If $\alpha(M)+Ker\beta = M$, $\beta \in End(M)$ then $(\alpha-\gamma)(m) \in Ker\beta$ for an automorphism of M and for all $m \in M$. For a pseudo-semi-projective module M, this notion is equivalent to uniquely generated module which was defined in [1].

MSC 2010: 16U60, 16U99 16E50, 16L30 19A13.
Keywords: Stable range, annihilator-stable rings, uniquely generated modules, von Neumann regular rings, unit-regular rings, matrix rings, pseudo-semi-projective module, kernel-stable modules

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Generalized class of boundary value problems with a constant retarded argument

ERDOĞAN ŞEN

Tekirdağ Namık Kemal University, Tekirdağ, Turkey

email: erdogan.math@gmail.com

In this study we shall find asymptotic formulas of eigenvalues and eigenfunctions for the eigenvalue problem $L := L\left(q; a, \lambda, r; h, H, d_j\right)$ $(j = 1, 2, 3)$ which consists of Sturm-Liouville equation

$$-y''(x) + q(x)y(x-a) = \lambda^2 r(x)y(x) = 0$$

on $\Lambda = \cup\Lambda^{\pm}$ with boundary conditions

$$y'(0) - hy(0) = 0,$$
$$y'(T) + Hy(T) = 0$$

and transmission conditions

$$y(c+0) = d_1 y(c-0),$$
$$y'(c+0) = d_2 y'(c-0) + d_3 y(c-0)$$

where $r(x) = \frac{1}{r_1^2}$ for $x \in \Lambda^- = [0, c)$ and $r(x) = \frac{1}{r_2^2}$ for $x \in \Lambda^+ = (c, T]$; the real-valued function $q(x)$ is continuous in Λ and has a finite limit $q(c \pm 0) = \lim_{x \to c \pm 0} q(x)$, $x - a \geq 0$, if $x \in \Lambda^-$; $x - a \geq c$, if $x \in \Lambda^+$; λ is a real spectral parameter; a, r_i $(i = 1, 2)$, h, H, d_j $(j = 1, 2, 3)$ are arbitrary real numbers such that $r_1 r_2 d_1 d_2 \neq 0$ and $d_1 r_2 = d_2 r_1$.

MSC 2010: 34K10, 34L20, 35R10
Keywords: Differential equation with retarded argument, transmission conditions, asymptotics of eigenvalues and eigenfunctions

Dual pole indicatrix curve and surface

SÜLEYMAN ŞENYURT[1], ABDUSSAMET ÇALIŞKAN[2]

[1,2] Ordu University, Ordu, Turkey

emails: [1]senyurtsuleyman@hotmail.com; [2]abdussamet65@gmail.com;

In this paper, the vectorial moment of the unit Darboux vector, which consists of the motion of the Frenet vectors on any curve, is reexpressed in the form of Frenet vectors. According to the new version of C^* vector, the parametric equation of the ruled surface corresponding to the unit dual pole indicatrix curve is given. The integral invariants of the closed ruled surface are rederived and illustrated by presenting with examples.

MSC 2010: 14H45, 14H50, 53A04
Keywords: Darboux vector, ruled surface, dual pole indicatrix curve, vectorial moment

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Curves and ruled surfaces according to alternative frame in dual space

SÜLEYMAN ŞENYURT[1], ABDUSSAMET ÇALIŞKAN[2]

[1,2] Ordu University, Ordu, Turkey

emails: [1]senyurtsuleyman@hotmail.com; [2]abdussamet65@gmail.com;

In this paper, the vectorial moments of the alternative vectors are expressed in terms of altrenative frame. According to the new versions of these vectorial moments, the parametric equations of the closed ruled surfaces corresponding to the $(\widehat{N})$, $(\widehat{C})$, $(\widehat{W})$ dual curves are given. The integral invariants of the these surfaces are computed and illustrated by presenting with examples.

MSC 2010: 14H45, 14H50, 53A04
Keywords: Alternative frame, closed ruled surface, vectorial moment, distribution parameter, Gauss curvature, dual angle of pitch, viviani's curve.

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Schrödinger equation with potential vanishing exponentially fast

TANFER TANRIVERDI

Harran University, Şanlıurfa, Turkey

e-mail: ttanriverdi@harran.edu.tr

Explicit solutions of differential equation

$$y'' + \left(\lambda + 20\,\mathrm{sech}^2\,x\right) y = 0y$$

and its eigenvalues are obtained by calculating complex residues. Eigenfunction expansions for this differential equation are also explored [1, 2, 3, 4, 5, 6, 7, 8].

MSC 2010: 35A24, 35B05, 35B24, 32A27
Keywords: Sturm-Liouville, Schrdinger equation, complex residues, explicit solutions
Acknowledgement: The Author thanks to HÜBAK (Scientific Research Council of Harran University)

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Blow up of solutions for a stochastic Klein-Gordon equation

HATICE TASKESEN

Van Yuzuncu Yil University, Van, Turkey

email: haticetaskesen@yyu.edu.tr

The Klein-Gordon equation is the first relativistic equation in quantum mechanics for the wave function of a particle with zero spin. It occurs in the study of various problems of mathematical physics such as general relativity, plasma physics, nonlinear optics, radiation theory, fluid mechanics, and was investigated in many papers [1], [2], [3], [4]

In the present work, we investigate the effect of stochastic terms on the explosion of solutions for a stochastic Klein-Gordon equation. By using a differential inequality and an energy inequality we prove that solutions of the problem blow-up in a finite time.

MSC 2010: 60H15, 35B44, 35Q40
Keywords: Stochastic Klein-Gordon equation, blow-up, energy inequality

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On the blow-up of solutions for a stochastic Camassa-Holm equation

HATICE TASKESEN[1], MOHANAD ALALOUSH[2]

[1,2]Van Yuzuncu Yil University, Van, Turkey

emails: [1]haticetaskesen@yyu.edu.tr; [2]alaloush.mohanad@gmail.com

In fluid dynamics, the Camassa-Holm (CH) equation is an integrable, bi-Hamiltonian model, which is proposed to explain the one-way propagation of shallow water waves on a flat bottom [1], [2] . However, it is difficult to include all the external effects in the deterministic equation, for example, the bottom of fluid may not be so flat or there will be some environmental noises. Therefore, a stochastic term must be added to the equation [3], [4]. In this work, the stochastic modified Camassa-Holm (SCH) equation is considered. Conditions that guarantee blow up of solutions in finite time are given.

MSC 2010: 60H15, 35B44, 35Q53

Keywords: Stochastic modified Camassa-Holm equation, blow-up, integrable equations

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Caristi type related fixed point theorems in two metric spaces

MUSTAFA TELCİ

Trakya University, Edirne, Turkey

email: mtelci@trakya.edu.tr

Let (X, d) and (Y, ρ) be metric spaces, let T be a mapping of X into Y and let S be a mapping of Y into X. It is proved that if there exists $x_0 \in X$ such that (X, d) and (Y, ρ) are related comlete and

$$\max\{d(x, STx), \rho(y, TSy)\} \leq \varphi(x) - \varphi(STx) + \psi(y) - \psi(TSy)$$

for all x in $R_X(x_0)$ and y in $R_Y(x_0)$, where $\varphi : X \to [0, \infty)$ and $\psi : Y \to [0, \infty)$, then

1. $\lim_{n\to\infty} x_n = \lim_{n\to\infty}(ST)^n x_0 = z$ and
   $\lim_{n\to\infty} y_n = \lim_{n\to\infty} T(ST)^{n-1}x_0 = w$ exist.

2. $Sw = z$ and $Tz = w$ if and only if $F : X \times Y \longrightarrow [0, \infty)$, $F(x, y) = d(x, Sy)$ and $G : X \times Y \longrightarrow [0, \infty)$, $G(x, y) = \rho(y, Tx)$ are w.l.s.c. at (z, w).

Further, if (2) holds, then $STz = z$ and $TSw = w$.

Some related fixed point results in two metric spaces are also derived by considering this result.

MSC 2010: 47H10, 54H25
Keywords: Fixed point, related comlete, weak lower semi-continuous

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Krasnoselskii fixed point theorem for singlevalued operators and multivalued operators

CESİM TEMEL

Van Yuzuncu Yil University, Van, Turkey

email: cesimtemel@yyu.edu.tr

In this study, we give some results of Krasnoselskii fixed point theorem for singlevalued operators and multivalued operators under weak topology in Banach spaces. In particular, we present the solutions of nonlinear operator equation

$$u = L(u) + S(u), \quad u \in U,$$

where L and S are weakly sequentially continuous operators. We also establish the existence of the solutions of inclusions of the form

$$u \in L(u) + S(u), \quad u \in U,$$

where L is based on the generalized D-Lipschitzian, $I - L$ may not be injective and S has weakly sequentially closed graph.

MSC 2010: 34K13, 47H04, 47H10
Keywords: Fixed point theorem, Krasnoselskii fixed point theorem, weakly sequentially continuous operator, multivalued operator

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Crossed modules of group-groupoids and double group-groupoids

SEDAT TEMEL[1], TUNCAR SAHAN[2], OSMAN MUCUK[3]

[1]Recep Tayyip Erdogan University, Turkey
[2]Aksaray University, Turkey
[3]Erciyes University, Turkey

emails: [1]sedat.temel@erdogan.edu.tr; [2]tuncarsahan@aksaray.edu.tr; [3]mucuk@erciyes.edu.tr

The notion of a crossed module which is initially introduced by Whitehead in [4] is related to the second relative homotopy groups for topological spaces. The categorical equivalence of crossed modules and group-groupoids which are also known in literature as *2-groups* was proved by Brown and Spencer in [2]. This categorical equivalence has also been extended by Porter in [3, Section 3] to a more general algebraic categories called categories of groups with operations. Double groupoids which can be thought as a groupoid objects in the category of groupoids are very useful for the proof of 2-dimensional Seifert-van-Kampen Theorem.

In this study we define a double group-groupoid to be a group object in the category of double categories and prove that these types of double groupoids are categorically equivalent to the crossed modules of group-groupoids and crossed square of groups.

MSC 2010: 20L05, 22A22, 18D35.
Keywords: Double category, crossed module, group-groupoid

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An optimality condition for non-smooth convex problems via *-subgradient

ALİ HAKAN TOR

Abdullah Gul University, Kayseri, Turkey

email: hakan.tor@agu.edu.tr

There are several optimality conditions for non-smooth unconstrained convex optimization problems via several types of subgradients. While some of them are being used by some researcher, all of them are needed according to the type of problem. In other words, this variety arises from the structure of optimization problems. In this work, a new optimality condition for the non-smooth convex optimization problem is given by using *-subgradient. *-subgradient is a new concept based on logarithmic differentiation. Actually, however logarithmic differential is valid for smooth function, *-subgradient can be used a non-smooth function, not necessarily differentiable.

MSC 2010: 49K99, 46N10, 47N10
Keywords: Optimality conditions, non-smooth convex problem, convex analysis

On the orbit surface of two parameter motion

FATİH TUĞRUL[1], ŞENAY BAYDAŞ[2], BÜLENT KARAKAŞ[3]

[1,2,3]Van Yuzuncu Yil University, Van, Turkey

emails: [1]fatihtugrul@yyu.edu.tr; [2]senay.baydas@gmail.com; [3]bulentkarakas@gmail.com

A displacement has that rotation axis is tangent, binormal, normal, Darboux vector, etc of a curve and translation vector with the knowledge of the curve is special displacement with two parameters. Image of a point under a displacement with two parameters is a surface. In this study, kinematics structures and properties of two parameters displacement are examined.

MSC 2010: 16H05, 70E15, 11E88
Keywords: Cliford algebra, 2 parameter motion, orbit surface

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Instability in nonlinear functional differential equations of higher order

CEMİL TUNÇ

Van Yuzuncu Yil University, Van, Turkey

email: cemtunc@yahoo.com

The author gives sufficient conditions for instability of solutions of some nonlinear delay differential equations of higher order. The technic of the proofs is based on the construction of suitable Lyapunov functionals. Some examples are given to illustrate the results obtained. The results of this paper improve some recent results can found in the literature.

MSC 2010: 39B82, 34K06
Keywords: differential equation, stability, higher order

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A note on certain qualitative properties of solutions in Volterra integro-differential equations

OSMAN TUNÇ

Van Yuzuncu Yil University, Van, Turkey

email: osmantunc89@hotmail.com

Volterra integral and integro differential equations with and without delay are essential tools in sciences and many fields. Because of this reality, in the recent years the qualitative behaviors of solutions of differential, integral and integro differential equations have extensively been discussed and are still being investigated by numerous authors. In this work, we consider three nonlinear Volterra integro-differantial equations. We investigated boundedness and exponantial stability of the solutions by Lypanuov functional. The results improve some the results that can found in literature.

MSC 2010: 45D05, 45M10, 45Jxx
Keywords: Volterra integro-differential equation, first order, exponantial stability, boundedness, Lyapunov function

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Galois theory and palindromic polynomials

FATMA TUTAR[1], ŞENAY BAYDAŞ[2], BÜLENT KARAKAŞ[3]

[1,2,3] Van Yuzuncu Yil University, Van, Turkey

emails: [1]ftutar94@gmail.com; [2]senay.baydas@gmail.com; [3]bulentkarakas@gmail.com

In this paper, Galois theory, field extension, palindromic polynomial, Galois groups are given. We introduce relation between Galois group and the roots of palindromic polynomials.

MSC 2010: 11R32, 12F10, 08A40
Keywords: Field extension, Galois group, Palindromic polynomials

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Canonical finite Blaschke products and decomposibility

SUMEYRA UCAR[1], NİHAL YILMAZ ÖZGÜR[2]

[1,2]Balıkesir University, Balıkesir, Turkey

emails: [1]sumeyraucar@balikesir.edu.tr; [2]nihal@balikesir.edu.tr

A canonical Blaschke product of degree n is a function of the following form:

$$B(z) = z\prod_{k=1}^{n-1}\frac{z-a_k}{1-\overline{a_k}z},$$

where a_k are in the unit disc for $1 \leq k \leq n-1$. In this study, we discuss a canonical finite Blaschke product B can be written as $B = B \circ M$ where M is a Möbius transformation different from identity using the nonzero zeros of B. Also, we investigate when such Blaschke products is composition of Blaschke products of lower degree.

MSC 2010: 30J10, 30D05, 51M15
Keywords: Finite Blaschke products, unit disc, composition of finite Blaschke products

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Applicability of regression analysis on the oxygen enriched combustion of Kutahya-Tuncbilek lignite

OZLEM UGUZ [1], ALI DEMIRCI[2], HANZADE HAYKIRI ACMA[3], SERDAR YAMAN[4]

[1]Marmara University, Istanbul, Turkey
[2,3,4]Istanbul Technical University, Istanbul, Turkey

emails: [1]ozlem.uguz@marmara.edu.tr; [2]demircial@itu.edu.tr; [3]hanzade@itu.edu.tr; [4]yamans@itu.edu.tr

Coal is the primary fossil fuel for energy generation. However, combustion of coal causes emission of many pollutant gases and particles to the atmosphere. Thus, clean coal technologies have been developed to minimize the hazards of burning coals in thermal power plants. Among these technologies, oxygen enriched combustion is one of the most commonly applied ones to low quality coals (lignites) of Turkey. In oxygen enriched combustion, lignites are burnt under the atmospheres of elevated oxygen concentrations. Oxygen enriched combustion increases available heat, improves ignition characteristics, reduces exhaust gas volume and increases energy efficiency [1, 2]. In this study, ground Kutahya-Tuncbilek lignite samples were burnt in a horizontal tube furnace at $200°C$, $450°C$ and $800°C$ under the atmospheres having the shares of $21\%O_2 + 79\%N_2$, $30\%O_2 + 70\%N_2$, $40\%O_2 + 60\%N_2$, $50\%O_2 + 50\%N_2$. After combustion tests, amount of carbon percentages in the burnout lignite sample was obtained by elemental analysis. Dependency of the carbon amount % on the furnace temperature and the oxygen share inside the furnace was investigated by applying 5 different regression models: $y = x_1 + x_2 + 1$ (Model1), $y = x_1^2 + x_2^2 + x_1 * x_2 + x_1 + x_2 + 1$ (Model 2), $y = x_1 * x_2 + x_1 + 1$ (Model 3), $y = x_1 * x_2 + x_2 + 1$ (Model 4), $y = x_1^2 + x_2^2 + x_1 + x_2 + 1$ (Model 5). Here x_1, x_2 and y stood for the furnace temperature, oxygen share inside the furnace and carbon amount %, respectively. Predicted equations, RMSE values, p-values, R^2 values were obtained for each regression model. It was seen that Model2. It was seen that Model2 gave the best fit to the data with the values of $R^2 = 0.985$, $RMSE = 0.037$ and p-value$= 2.06 * 10^{-5}$. Thus, the required oxygen/nitrogen ratio and the furnace temperature could be calculated to burn Kutahya-Tuncbilek lignite in the most effective way without the necessity of experimental work.

MSC 2010: 62J05, 80A25, 00A69
Keywords: Regression analysis, coal, oxygen enriched combustion, energy

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Applicability of regression analysis on the oxygen enriched combustion of Adiyaman-Golbasi lignite

OZLEM UGUZ [1], ALI DEMIRCI[2], HANZADE HAYKIRI ACMA[3], SERDAR YAMAN[4]

[1]Marmara University, Istanbul, Turkey
[2,3,4]Istanbul Technical University, Istanbul, Turkey

emails: [1]ozlem.uguz@marmara.edu.tr; [2]demircial@itu.edu.tr; [3]hanzade@itu.edu.tr; [4]yamans@itu.edu.tr

Coal is predicted to cover about almost one third of the global energy needs in the year of 2021. Because of the high share of coal in energy industry, clean coal technologies have been developed by scientists. Oxygen enriched combustion is one of these technologies as it reduces emissions and increases efficiency [1, 2]. In this study, oxygen enriched combustion was applied to ground low quality Adiyaman- Golbasi lignite at temperatures of $200°C$, $400°C$ and $600°C$ in the horizontal tube furnace, under the atmospheres with the shares of $21\%O_2+79\%N_2$, $30\%O_2+70\%N_2$, $40\%O_2+60\%N_2$, $50\%O_2+50\%N_2$. The carbon amounts % of the burnt samples were obtained by elemental analysis. Regression analysis was performed to the data to verify the carbon amount % left in the burnout sample when the combustion temperature and the oxygen/nitrogen ratio were given. Tested models for regression analysis were as follows: $y = x_1 + x_2 + 1$ (Model1), $y = x_1^2 + x_2^2 + x_1 * x_2 + x_1 + x_2 + 1$ (Model 2), $y = x_1 * x_2 + x_1 + 1$ (Model 3), $y = x_1 * x_2 + x_2 + 1$ (Model 4), $y = x_1^2 + x_2^2 + x_1 + x_2 + 1$ (Model 5). In these equations, x_1, x_2 and y represented the furnace temperature, oxygen share inside the furnace and carbon amount %, respectively. As a result of the analysis it was found that Model2 gave the most accurate results, since it gave the values of $R^2 = 0.981$, $RMSE = 0.0465$ and p-value= $4.59 * 10^{-5}$. Thus, by applying Model2, required values of combustion temperature and oxygen/nitrogen ratio can be predicted to burn all of the carbon content in Adiyaman- Golbasi lignite.

MSC 2010: 62J05, 80A25, 00A69
Keywords: Regression analysis, coal, oxygen enriched combustion, energy

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On uniformly pr-ideals in commutative rings

RABIA NAGEHAN UREGEN

Erzincan Binali Yildirim University, Erzincan, Turkey.

email: rabia.uregen@erzincan.edu.tr

Let R be a commutative ring with nonzero identity and I be a proper ideal of R. Then I is said to be a uniformly pr-ideal if there exists $N \in \mathbb{N}$ such that $ab \in I$ with $ann(a) = 0$ implies $b^N \in I$. In that case the smallest integer $N \in \mathbb{N}$ is called the order of I and denoted by $ord_R(I) = N$. Among many results in this presentation, it is given some characterizations and properties of this new classes of ideals similar to prime ideals. Furhermore, it is investigated that the relations between uniformly pr-ideals and some classical ideals such as r–ideals, uniformly primary ideals and strongly primary ideals.

MSC 2010: 13A15, 13E05
Keywords: r-ideal, pr-ideal, primary ideal, uniformly primary ideal

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On 2-absorbing ideals

REZVAN VARMAZYAR

Islamic Azad University, Khoy, Iran

email: varmazyar@iaukhoy.ac.ir

Let R be a commutative ring with identity. A proper ideal A of R is called a 2-absorbing ideal if $xyz \in A$ for $x, y, z \in R$ implies that $xy \in A$ or $xz \in A$ or $yz \in A$. In this article, some properties of 2-absorbing ideals are given.

MSC 2010: 13C99
Keywords: Prime ideal; n-absorbing ideal; 2-absorbing ideal.

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Studying the kinetic parameters and mechanism of the thermal decomposition (dehydration, dehydroxylation and decarbonylation) of some clays using TG traces

ZEKİ YALÇINKAYA[1], ŞENOL KUBİLAY[2], ALİ SAVRAN[3], NECLA ÇALIŞKAN[4]

[1,2,3,4] Van Yuzuncu Yil University, Van, Turkey

emails: [1]zeki@yyu.edu.tr; [2]senolkubilay@yyu.edu.tr; [3]alisavran@yyu.edu.tr; [4]ncaliskan@hotmail.com

Two methods, Reich-Stivala (R-S) and double-log (D-L), were employed to distinguish one mechanism from among 15 theoretical possibilities for heterogeneous solid-state reactions using TG data [1-3]. As known, the rate of reaction of thermal decomposition of solids can be expressed by general equation

$$\frac{d\alpha}{dt} = Ae^{-E/RT} f(\alpha) = Ae^{-E/RT}(1-\alpha)^n \quad (1)$$

Where $f(\alpha)$ is a function which depends on the reaction mechanism, A=frequency factor, T=temperature (K), =conversion, n= reaction order. From eq (1), following eq (2) can readily be obtained (the R-S expression).

$$log[\frac{g(\alpha_1)}{\alpha_2}\frac{T_1T_2}{(T_1-T_2)}] = \frac{E}{2,303R} \quad (2)$$

From eq (2), the double-log expression can be derived

$$\frac{log[\frac{T_R^2}{g(\alpha_R)}(\frac{g(\alpha_1)}{T_1^2})]}{log[\frac{T_R^2}{g(\alpha_R)}(\frac{g(\alpha_2)}{T_2^2})]}\frac{T_1(T_R-T_2)}{T_2(T_R-T_1)} = Z_1 \quad (3)$$

Where T_R, α_R denote an arbitrary reference temperature and its corresponding conversion, respectively. Thermal gravimetric (TG) curves of clays (Bardaki, Tilkitepe from the Van region, Turkey) were determined. Deformations are defined here as changes of the clay by dehydration, dehydroxylation, and decarbonylation. Equations (2 and 3) were employed to determine the mechanisms of thermal decomposition of the clays. Activation energies related to the deformations calculated from the TG curves and the reaction order are also determined. All the calculations were made with EXCEL and recorded as EXCEL macro, then were used.

MSC 2010: 80A50, 00A69
Keywords: Thermogravimetry, kinetics, mechanism, Excel Macro

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Clay transition (dehydration, dehydroxylation and decarbonylation) kinetics by DTA

ZEKİ YALÇINKAYA[1], ALİ SAVRAN[2], ŞENOL KUBİLAY[3], NECLA ÇALIŞKAN[4]

[1,2,3,4]Van Yüzüncü Yıl University, Van, Turkey

emails: [1]zeki@yyu.edu.tr; [2]alisavran@yyu.edu.tr; [3]senolkubilay@yyu.edu.tr, [3]ncaliskan@hotmail.com

The effects of the kinetics of reactions of the type solid solid + gas on the corresponding differential thermal analysis traceses are studied. When a reaction occurs in DTA, the change in the thermal properties of the sample is indicated by a deflection, or peak [1]. The information so obtained is used to analyze the differential thermal patterns of clays. Two samples of the clays were chosen from Bardakçi, Tilkitepe, the Van region, Turkey. The peaks are defined here as changes of the clay by dehydration, dehydroxylation, and decarbonylation.

Reich [2] have developed a method to obtain an approximate expression for the energy of activation and reaction order. We used the method (Eq 1) for determining the kinetic parameters from the shape of the differential thermal analysis peak.

$$\left(\frac{T_1}{T_2}\right)^2\left[\frac{\Delta T_1}{\Delta T_2}\right]=F\left(T\right)=\left(\frac{\overline{a}_{T,1}}{\overline{a}_{T,2}}\right)^n\left[\frac{1-\left(\frac{\overline{a}_{T,1}}{A_T}\right)^{1-n}}{1-\left(\frac{\overline{a}_{T,2}}{A_T}\right)^{1-n}}\right],\ n\neq 1 \tag{1}$$

Where, T=tempreture (K), ΔT=height of the DTA curve from the baseline; n=order of reaction, A_T=total thermogram area and $\overline{a}=\int_T^\infty \Delta T dT$.

One of previously reported algorithm [3] for the estimation of kinetic parameters, activation energy (E) end reaction order (n), from DTA traces assuming an n-type mechanism, after transformed the algorithm to EXCEL VBA macro, were used.

MSC 2010: 80A50, 00A69
Keywords: Differential thermal analysis, kinetics, mechanism, Excel Macro

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On the weighted pseudo almost periodic solutions of nonlinear functional Nicholson's blowflies equations

RAMAZAN YAZGAN[1], CEMİL TUNÇ[2]

[1,2] Van Yuzuncu Yil University, Van, Turkey

emails: [1]ryazgan503@gmail.com; [2]cemtunc@yahoo.com

In this paper, we prove the existence and the global exponential stability of the unique weighted pseudo almost-periodic solution of a class generalized Nicholson's blowflies model with a linear harvesting term and mixed delays. Some sufficient conditions are given for the existence and the global exponential stability of equation considered by using point theorem and differential inequality techniques. The results of this paper complement the previously known ones. Finally, an illustrative example is given to demonstrate the effectiveness of our results.

MSC 2010: 34K14,34K20,92D25
Keywords: Weighted pseudo almost periodic solution, Nicholson's Blowflies model, Fixed point.

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A new generalization of Szász operators and its approximation properties

SERDAL YAZICI[1], BAYRAM ÇEKİM[2]

[1,2]Gazi University, Ankara, Turkey

e-mails: [1]serdal.yazici@gazi.edu.tr ; [2]bayramcekim@gazi.edu.tr

In this presentation, we attempt to introduce a new generalization of the Szàsz operators including Hermite polynomials with two variable and shed light on their approximation features with help of the classical modulus of continuity, Peetre's -K functional, the class of Lipschitz functions, second modulus of continuity, Voronovskaya type asymptotic formula for these operators.

MSC 2010: 41A25, 41A35
Keywords: Hermite polynomials, modulus of continuity, Lipschitz continuity, approximation theory, Voronovskaya type aymptotic formula.

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Characterization of regular morphisms in terms of abelian categories

TÜLAY YILDIRIM

Gebze Technical University, Kocaeli, Turkey

email: tyildirim@gtu.edu.tr

In this talk, we focus on the regular morphisms in abelian categories which asserts that a morphism $f : M \to N$ in an abelian category $\mathbb{C}$ is called regular if there exists a morphism $g : N \to M$ such that $f = fgf$. In the present paper, we establish some further results involving regular morphisms in abelian categories which extend known properties for modules and prove a property on regular compositions of morphisms. We generalize from modules to abelian categories the concept of consecutive pair of morphisms, recently introduced by Facchini and Leroy [1]. Inspired by Ara, Goodearl, OMeara and Pardo [3], equivalent morphisms in abelian categories are taking into the consideration. Moreover, we consider one-sided unit regular morphisms in categories, based on Ehrlichs concept of one-sided unit regular element in a ring [2]. In this talk, after a brief introduction to the subject is discussed, a recent result with a joint work S. Crivei M. Tamer Koşan and Tülay Yıldırım [4] among these lines will be presented.

MSC 2010: 34B05, 34A08
Keywords: Abelian category, regular morphism, consecutive pair of morphisms, equivalent morphisms, unit regular morphism.
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Statistical inference for the inverse Weibull distribution

ASUMAN YILMAZ[1], MAHMUT KARA[2]

[1,2]Van Yuzuncu Yil University, Van, Turkey

emails: [1]asumanduva@gmail.com; [2]mkara2581@gmail.com

The inverse Weibull distribution is known as type 2 extreme value or Frechet distribution. This distribution has been used to model, many real life applications such as degradation of mechanical components, engineering, hydrological and ecological.

$$f(x) = \frac{\alpha}{\beta}(\frac{x}{\beta})^{-(\alpha+1)}e^{-(x/\beta)^{-\alpha}},\ \ x > 0,\ \ \alpha, \beta > 0$$

and

$$F(x) = e^{-(x/\beta)^{-\alpha}},\ \ x > 0,\ \ \alpha, \beta > 0,$$

respectively. In this study, parameter estimation of inverse Weibull distribution are given by using the methods such as maximum likelihood, L- moment, least square, weighted least square, approximate Bayesian estimates with Lindleys method. Furthermore the performances of the obtained estimators are compared with respect to their biases, log-likelihood values, Q-Q plots, the density plots, distribution plots, distribution plots, AIC, BIC and mean square errors through a simulation study. At the end of study, the procedure is illustrated based on real data.

MSC 2000: 60J10, 60J22, 11K06
Keywords: Inverse Weibull distribution, maximum likelihood method, Lindley method.

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Existence of solutions for a system of coupled fractional boundary value problems

FULYA YORUK DEREN

Ege University, Izmir, Turkey

email: fulya.yoruk@ege.edu.tr

In this study, we are interested in the existence and multiplicity of positive solutions for a system of fractional differential equations subject to Riemann Stieltjes integral boundary conditions. Our analysis is based upon some theorems from fixed point theory. Finally, an example is given to illustrate our main result.

MSC 2010: 34B10, 34B18, 39A10.
Keywords: Multiple positive solution, fractional differential equation, fixed point theorem.
Acknowledgement: This work is supported by Ege University, Scientific Research Project (BAP), Project Number: 2015 FEN 069.

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Input distinguishability of linear dynamic control systems

AWAIS YOUNUS[1], HONGWEI LOU[2]

[1,2] Fudan University, Shanghai, China
[1] Bahauddin Zakariya University, Multan, Pakistan

email: awais@bzu.edu.pk; hwlou@fudan.edu.cn

When observabilities of hybrid dynamic systems are considered, the distinguishability of subsystems takes a very important role. Necessary and sufficient conditions for distinguishability of linear dynamic systems are obtained. The main result state that the input distinguishability of dynamic control systems is equivalent to nontrivial zero dynamics of the systems.

MSC 2010:
Keywords: Distinguishability, hybrid dynamic systems, zero dynamics

Abstracts of posters

On Caputo and Riemann-Liouville fractional-order derivatives with fixed memory length

MOHAMMED SALAH ABDELOUAHAB[1], SAFA BOURAFA[2]

[1,2]Mila University center, Mila, Algeria

email: [1] medsalah3@yahoo.fr

Recently some researchers have demonstrated that the fractional-order derivative of a non-constant periodic function is not a periodic function with the same period [1, 3, 2, 4, 5]. As a consequence of this property the time-invariant fractional order systems does not have any non-constant periodic solution unless the lower terminal of the derivative is $\pm\infty$, which is not practical. This property limits the applicability of the fractional derivative and makes it unfavorable for periodic real phenomenon [6]. Therefore, enlarging the applicability of fractional systems to such periodic real phenomenon is an important research topic. In this work, we extend the modification of the Grünwald-Letnikov definition of fractional derivative introduced in [6] to the Caputo and Rieman-Liouville fractional-order derivatives. This modification consists of fixing the memory length and varying the lower terminal of the derivative. It is shown that the modified definition of fractional derivative preserves the periodicity.

MSC 2010: 34A08, 34A12, 37C27
Keywords: Fractional-order derivative, memory length, periodic solution

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On approximation by generalized Bernstein-Durrmeyer operators

ECEM ACAR[1], AYDIN İZGİ[2]

[1,2]Harran University, Şanlıurfa, Turkey

[1]karakusecem@harran.edu.tr; [2]a-izgi@harran.edu.tr

The purpose of this paper is to introduce the generalized Bernstein-Durrmeyer type operators and obtain some approximation properties of these operators studied in the space of continuous functions of two variables on a compact set. The rate of convergence of these operators are given by using the modulus of continuity. The degree of approximation for the Lipschitz class of functions and the Voronovskaja type asymptotic theorem are studied and some differential properties of these operators are proved. Furthermore, the convergence of the operators by illustrative graphics in Maple to certain functions for two dimensional cases are given.

MSC 2010: 41A10, 41A25, 41A36
Keywords: Approximation, Bernstein-Durrmeyer Operators, modulus of continuity, rate of convergence, Voronovskya type theorem

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A fuzzy methodology on surface representation of greenhouse gas estimation

FİLİZ KANBAY[1], NURTEN VARDAR[2]

[1,2]Yildiz Technical University, Istanbul, Turkey

emails: [1]fkanbay@yildiz.edu.tr; [2]vardar@yildiz.edu.tr;

In this study, the greenhouse gas emissions emitted from transit ships passing through the Bosphorus are predicted by using fuzzy inference system. The effect of the cruising speed and gross tonnage changes of ships are taken into consideration in a fuzzy model. The results are given as surfaces.

MSC 2010: 65D18, 68T27, 03B52
Keywords: Greenhouse gas, ship, fuzzy, surface

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Asymptotic aspect of some functional equations

MOHAMMAD BAGHER MOGHIMI

University of Mohaghegh Ardabili, Ardabil, Iran

email: moghimi@uma.ac.ir

In this note we study the asymptotic stability behavior of some functional equations such as Jensen $2f(\frac{x+y}{2}) = f(x)+f(y)$, quadratic $f(x+y)+f(x-y) = 2f(x)+2f(y)$ and Drygas $f(x+y)+f(x-y) = 2f(x) + f(y) + f(-y)$ functional equations. We show that if a functional equation of these type functional equations holds approximately for large arguments with an upper bound, then it also valids approximately everywhere with a new upper bound which is a constant multiple of its bound.

MSC 2010: 39B82, 39B62
Keywords: Functional equatin, Jensen equation, quadratic equation, Drygas equation, asymptotic stability

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Existence of positive solutions for second order impulsive boundary value problems on the half-line

ILKAY YASLAN KARACA[1], AYCAN SİNANOĞLU ARISOY[2]

[1,2]Ege University, Izmir, Turkey

emails: [1]ilkay.karaca@ege.edu.tr; [2]aycansinanoglu@gmail.com;

This paper deals with the existence of positive solutions for a second-order impulsive boundary value problem on the half-line. Our existence result is based on a fixed point theorem and a compactness argument.

MSC 2010: 34B18, 34B37, 34B40
Keywords: Impulsive boundary value problem, infinite interval, fixed point theorem, positive solutions

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