The Lit-Only Sigma Game on a Simple Graph

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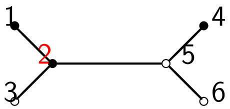
Lit-only sigma game

Let $X = (S, E)$ be a finite simple connected graph of order n. Every vertex of X can be assigned to either black state or white state to form a configuration. A move on a configuration is to select one vertex $s \in S$ having black state and then change those states of all neighbors of s. Given two configurations, the goal is to decide if one can reach the other by a sequence of moves. This is the lit-only sigma game on X.

Linear algebraic modeling

A configuration of the lit-only sigma game on the graph $X = (X, E)$ described in last page is naturally associated with a column vector u in the n-dimensional vector space F_2^n over F_2 $(n = |S|)$, where $u_i = 1$ iff the vertex $i \in S$ is black. Each move is then naturally associated with an $n \times n$ matrix in $\mathrm{GL}_n(F_2)$ that acts on F_2^n by left multiplication.

Example A



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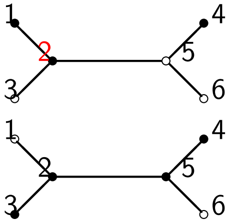

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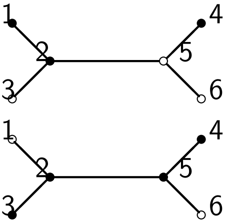

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} =$$

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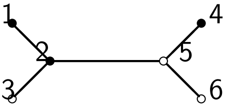

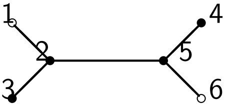

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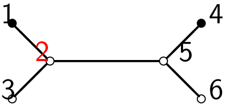
Example B



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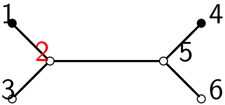

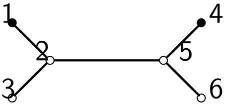

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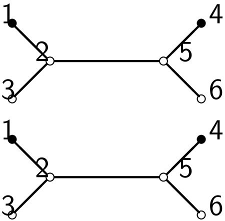

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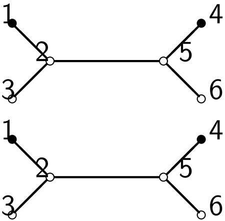

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History I

This game implicitly appeared in M. Chuah (蔡孟傑) and C. Hu's papers in 2004 when they studied the equivalence classes of Vogan diagrams. Gerald Jennhwa Chang (張鎮華) introduced this game to the Chinese combinatorists by a talk in the title "Graph Painting and Lie Algebra" in 2005 International and Third Cross-strait Conference on Graph Theory and Combinatorics. (2005 年圖論與組合學國際學術會議暨第三屆海峽兩岸圖論與組合學學術會議) It was considered as a new game and the name of this game was not given when Chang's talk was given.

History II

Xinmao Wang and Yaokun Wu recognized this game is a variety of anther game, called sigma game, which has been studied actively since 1980's. Even for the lit-only sigma game, M. Chuah and C. Hu were not the first two to study. It appears as early as in 2001 paper of H. Eriksson, K. Eriksson, J. SjAöstrand.

Flipping groups and flipping classes

Definition

Let $X = (X, E)$ be a graph. For a vertex $s \in S$, we associate a matrix $\mathbf{s} \in \mathrm{Mat}_n(F_2)$, denoted by the bold type of s, as

$$\mathbf{s}_{uv} = \begin{cases} 1, & \text{if } u = v, \text{ or } v = s \text{ and } uv \in E; \\ 0, & \text{else,} \end{cases}$$

where $u, v \in S$.

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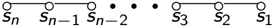
Definition

The orbits of F_2^n under $\mathbf{W}$ are called the flipping classes of X.

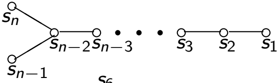
Dynkin diagram

Flipping classes of Dynkin Diagrams and extended Dynkin diagrams are determined by Meng-Kiat Chuah and Chu-Chin Hu in 2004, 2006 respectively.

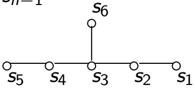
$A_n (n \geq 1)$



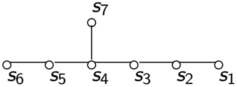
$D_n (n \geq 4)$



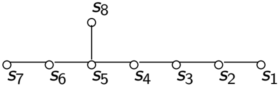
E_6



E_7



E_8



A graph with a long path

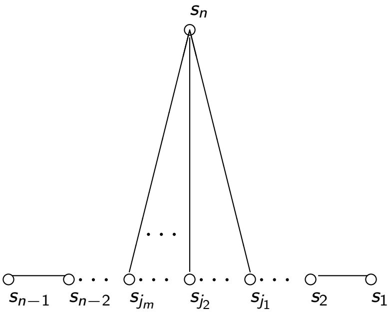


Figure: The graph $X = (S, E)$.

Notations 1

Let S be a connected graph with n vertices $s_1, s_2, \ldots, s_n$ that contains an induced path $s_1, s_2, \ldots, s_{n-1}$ of $n-1$ vertices, and s_n has neighbors $s_{j_1}, s_{j_2}, \ldots, s_{j_m}$ with $1 \leq j_1 < j_2 \cdots < j_m \leq n-1$. Let $\widetilde{s}_1, \widetilde{s}_2, \ldots, \widetilde{s}_n$ denote the characteristic vectors of F_2^n and let $\mathbf{s_1}, \mathbf{s_2}, \ldots, \mathbf{s_n}$ denote the flipping moves associated with $s_1, s_2, \ldots, s_n$ respectively.

Set

$$\overline{1} = \widetilde{s}_1, \; \overline{i+1} = \mathbf{s_i}\mathbf{s_{i-1}} \cdots \mathbf{s_1}\overline{1} \quad (1 \leq i \leq n-1), \quad \overline{n+1} := \widetilde{s}_n.$$

and consider the following three sets

$$\begin{aligned} \Pi &= \{\overline{1}, \overline{2}, \ldots, \overline{n}\}, \\ \Pi_0 &= \{\overline{i} \in \Pi \mid \; < \overline{i}, \widetilde{s}_n >= 0\}, \\ \Pi_1 &= \Pi - \Pi_0. \end{aligned}$$

Notations 2

By using the graph structure we can compute the following value

$$|\Pi_1| = \sum_{k=1}^{\lceil \frac{m}{2} \rceil} j_{2k} - j_{2k-1}.$$

Let

$$\Delta := \begin{cases} \Pi, & \text{if } |\Pi_1| \text{ is odd;} \\ \Pi \cup \{\overline{n+1}\} - \{\overline{n}\}, & \text{if } |\Pi_1| \text{ is even} \end{cases}$$

be the simple basis of F_2^n as shown in the beginning of Section **??**. For a vector $u \in F_2^n$ let $sw(u)$ denote the simple weight of u, i.e. the number nonzero terms in writing u as a linear combination of elements in Δ. Let U be the subspace spanned by the vectors in Π. For $V \subseteq F_2^n$ and $T \subseteq \{0, 1, \ldots, n\}$,

$$V_T := \{u \in V \mid sw(u) \in T\},$$

and for shortness $V_{t_1,t_2,\ldots,t_i} := V_{\{t_1,t_2,\ldots,t_i\}}$. Let *odd* be the subset of $\{1, 2, \ldots, n\}$ consisting of odd integers.

Notations 3

Set

$$\begin{aligned} A_i &= \{j \in [n] \mid j \equiv i, n + |\Pi_1| - i \pmod 4\}, \\ B_i &= \{j \in [n-1] \mid j \equiv i, i + |\Pi_1| - 2, n - i, n - i + |\Pi_1| - 2 \pmod 4\}, \\ C_i &= \{j \in [n] \mid j \equiv i, i + |\Pi_1|, n + 2 - i, n + 2 - i + |\Pi_1| \pmod 4\}. \end{aligned}$$

Let $\mathcal{P}$ denote the set of orbits of the flipping puzzle on S. Then the set $\mathcal{P}$ and its cardinality $|\mathcal{P}|$ are given in the following table according to the different cases of the pair $(|\Pi_1|, n)$ in the first two columns.

Flipping classes of a graph with a long path

$\lvert\Pi_1\rvert$	n	nontrivial $O \in \mathcal{P}$ (might be repeated)	$\lvert\mathcal{P}\rvert$
$3 \leq \lvert\Pi_1\rvert \leq n-3$, $\lvert\Pi_1\rvert$ is odd	even	U_{A_j}	3
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$4 \leq \lvert\Pi_1\rvert \leq n-3$, $\lvert\Pi_1\rvert$ is even	even	$U_{B_j}, \overline{U}_{C_j}$	6

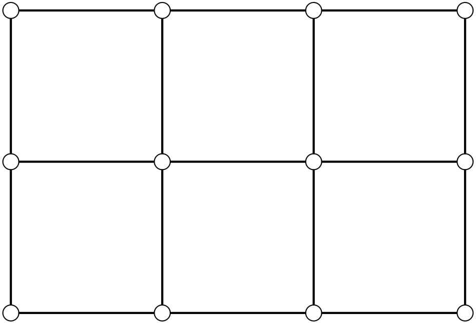
$4 \le \lvert\Pi_1\rvert \le n-3$, $\lvert\Pi_1\rvert$ is even	odd	$U_{B_j}, \overline{U}_{C_j}$	4
$\lvert\Pi_1\rvert = 1$		$U_{t,n+1-t}$	$\lceil (n+2)/2 \rceil$
$\lvert\Pi_1\rvert = 2$	even	$U_{i,n-i}, \overline{U}_{C_1}, \overline{U}_{C_2}$	$(n+6)/2$
$\lvert\Pi_1\rvert = 2$	odd	$U_{i,n-i}, \overline{U}_{C_1}, \overline{U}_{C_2}$	$(n+3)/2$
$\lvert\Pi_1\rvert = n-2$, $\lvert\Pi_1\rvert$ is odd	odd	U_{odd}, U_{2i}	$(n+3)/2$
$\lvert\Pi_1\rvert = n-2$, $\lvert\Pi_1\rvert$ is even	even	$U_{odd}, U_{2h,n-2h}$, $\overline{U}_{odd}, \overline{U}_{2g,n+2-2g}$	$(n+6)/2$
$\lvert\Pi_1\rvert = n-1$, $\lvert\Pi_1\rvert$ is odd	even	$U_{2t-1,2t}$	$(n+2)/2$
$\lvert\Pi_1\rvert = n-1$, $\lvert\Pi_1\rvert$ is even	odd	$U_{2h-1,2h,n-2h,,n+1-2h}$, $\overline{U}_{2g-1,2gn+2-2g,n+3-2g}$	$(n+3)/2$

Flipping classes of line graphs

Yaokun Wu, Lit-only sigma game on a line graph, European Journal of Combinatorics 30(2009), 84-95.

Problems

Determine the flipping classes of X when X is a chessboard.



Maximum-orbit-weight

For $u \in F_2^n$, let $w(u)$ denotes the Hamming weight of u, and for an flipping class O of X, $w(O) := \min\{w(u) \mid u \in O\}$ is called the weight of the flipping class O. The number

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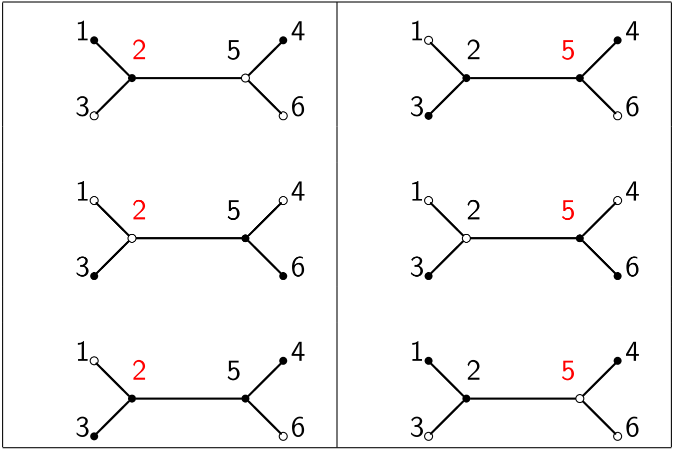
4. When X has a long path, we give a necessary and sufficient condition for $M(X) = 1$.

Six alternative moves 2, 5, 2, 5, 2, 5 of the edge 25



Coxeter group associated a graph

Let $X = (S, E)$ denote a simple connected graph with vertex set $\{s_1, s_2, \dots, s_n\}$.

Definition

The **Coxeter group** $W := W(X)$ of a simple connected graph $X = (S, R)$ is the group with the set $S = \{s_i \mid 1 \le i \le n\}$ of generators subject only to relations

$$\begin{aligned} s_i^2 &= 1, \\ (s_i s_j)^3 &= 1, \quad \text{if } ij \in E, \\ (s_i s_j)^2 &= 1, \quad \text{if } ij \notin E. \end{aligned}$$

Relation between Coxeter group and flipping group

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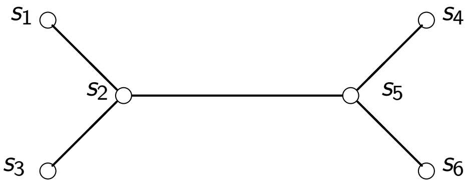
4. Among all n-vertex graphs containing an induced $(n-1)$-vertex path, there are at most $n-1$ flipping groups up to isomorphism.
5. If X is the line graph of a graph with m edges and n vertices, then the flipping group $\mathbf{W}$ of X isomorphic to $(\mathbb{Z}/2\mathbb{Z})^{(n-1)(m-n+1)} \rtimes S_n$ if n is odd; $(\mathbb{Z}/2\mathbb{Z})^{(n-2)(m-n+1)} \rtimes S_n$ if n is even.

Reeder's game

A **configuration** is an assignment of one of two color, black or white, to each vertex of X. A **move** applied on a configuration is to select a vertex v having an odd number of black neighbors and change the color of v.

Reeder's game

Let X be



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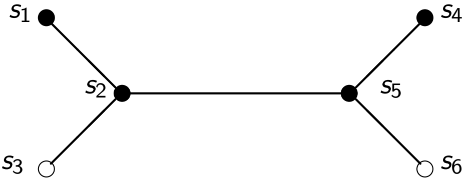

$$x = s_1 + s_2 + s_4 + s_5$$

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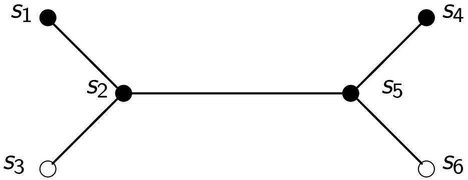

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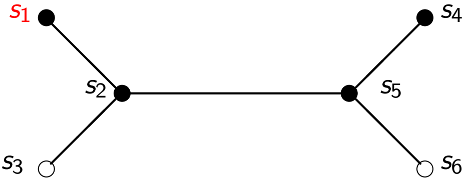

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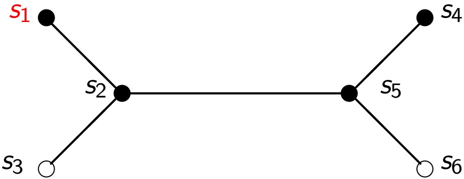

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$$\rho_1(x) = ?$$

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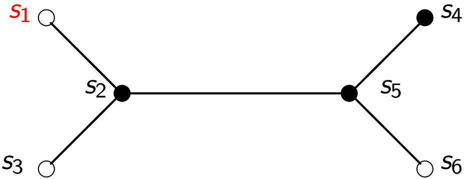

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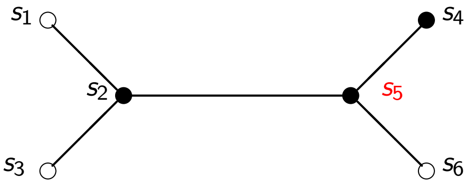

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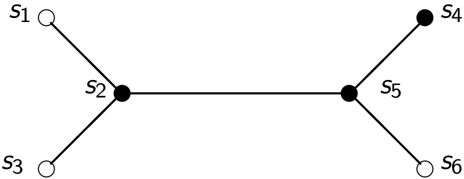
$$\rho_5(\rho_1(x)) = ?$$

Reeder's game



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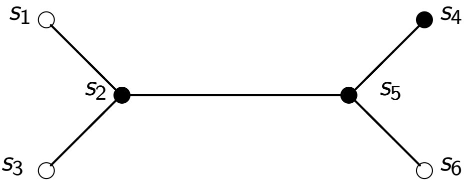

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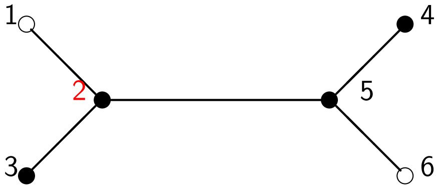
Duality between Reeder's game and lit-only σ-game



$$\begin{pmatrix}0\\1\\1\\1\\1\\0\end{pmatrix}^t = \begin{pmatrix}0\\1\\1\\1\\1\\0\end{pmatrix}^t \begin{pmatrix}1&1&0&0&0&0\\0&1&0&0&0&0\\0&1&1&0&0&0\\0&0&0&1&0&0\\0&1&0&0&1&0\\0&0&0&0&0&1\end{pmatrix} \begin{pmatrix}0\\1\\1\\1\\1\\0\end{pmatrix} = \begin{pmatrix}1\\1\\0\\1\\0\\0\end{pmatrix}$$

Duality

The orbits of Reeder's game are called Reeder's classes. A graph X is nonsingular if the determinant $det(A) = 1$ in F_2, where A is the adjacency matrix of X.

Lemma

Suppose that X is a nonsingular graph. Then there exists a bijection between flipping classes and Reeder's classes.

Reeder's Theorem

Theorem (2005, M. Reeder)

Suppose that X is a tree with a perfect matching, not a path. Then there are exactly three Reeder's classes on X.

Orbits distinguishing

Theorem (2009, J. Goldwasser, X. Wang, Y. Wu)

Suppose that X is a nonsingular graph of n vertices. Let $u \in F_2^n$ be a configuration with $u_i = 0$ for some i. Let A_i denote the i-th column of the adjacency matrix A. Then u and $u + A_i$ are in two different flipping classes.

Applications

By the dual connection between Reeder's game and lit-only σ-game, and using J. Goldwasser, X. Wang, Y. Wu's Theorem to distinguish flipping classes, Hau-wen Huang can show

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Problem: Find an algorithm to do this.

Generalization

Suppose that X is a nonsingular graph, not a line graph.

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Problem. Characterize the case $M(X) = 1$ when X is nonsingular.

Revisit the move on Reeder's game

Let $u \in F_2^n$ be a configuration of Reeder's game on $X = (S, E)$. Let **s** be the $n \times n$ move matrix associate with the vertex $s \in S$. We also use s to denote the characteristic vector of $s \in S$. Let $f_s(u)$ denote the new configuration from u by applying the move **s** in Reeder's game on X. Then

$$\begin{aligned} f_s(u)^t &= u^t \mathbf{s} \\ &= u^t + (u^t A s)\ s^t \\ &= u^t + < u, s > s^t, \end{aligned}$$

where $< u, s > := u^t A s$ is the inner product.

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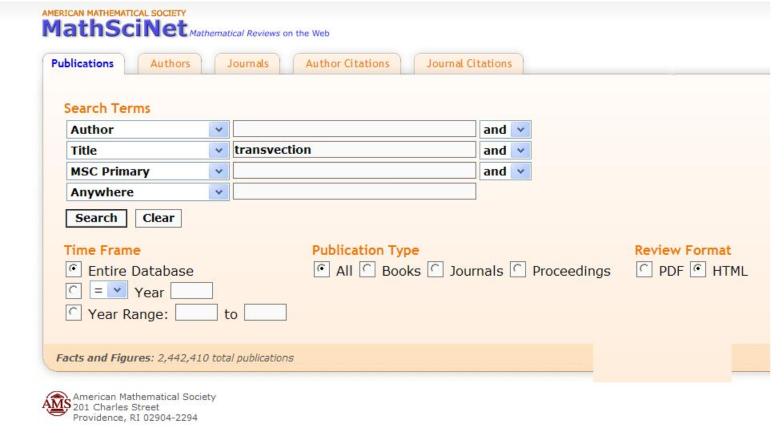
The above function f_s is called a transvection in the literature.



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