NCTU Department of Applied Mathematics

Qualifying Examination in Discrete Mathematics

for the Ph. D. Program September 2019

<u>**Note**</u>: The proofs and statements must be detailed. When you quote some theorems, please prove them.

1. Find the general solution of a linear recurrence relation $a_n=a_{n-1}+6a_{n-2}$ with $a_0=a_1=1$ by two different methods. (20%)

2. Let M be the set of all 2×5 matrices over {1,2,...,9} and S be the subset of M satisfying, for each 2×5 matrix A of S, every two elements in the same rows of A or the same column of A are different. Find the cardinality of S. (20%)

3. Assume that G is a simple graph and k is the length of a longest path in G. Prove that the chromatic number of G is less than or equal to k+1. (20%)

4. Allice plans to visit 37 towns in 25 days and she visits at least one new down in every day. Prove that she will visit exactly 12 towns in some period of consecutive days. (10%)

5. True or False. (If the statement is true, prove it; if it is false, give a counterexample) (10%×3)
   (a) If G is a planar graph with the girth at least 4, then G is 4-colorable.
   (b) There exists a 2-(12,5,3)-design.
   (c) If G is a simple planar bipartite graph of order n>3, then $|E(G)| \leq 2n-4$.