博士班資格考

Ph.D. Qualifying Examination in Numerical Analysis

Spring, 2011

Please write down all the detail of your computations and proofs.

(1) (15%) Use the finite difference method with second-order accuracy to approximate the solution

$$u''(x) = 2, \quad 0 < x < 1,$$
$$u(0) = 1, \ u(1) = 2.$$

Write down the associated linear system but do not need to solve it.

(2) Consider the elliptic partial differential equation:

$$\frac{\partial^2 u}{\partial x^2}(x,y) + \frac{\partial^2 u}{\partial y^2}(x,y) = 0 \ \text{ on } \ \Omega = \{(x,y); 0 < x < 0.5, \ 0 < y < 0.5\},$$
$$u(0,y) = 0, \ u(x,0) = 0, \ u(x,0.5) = 200x, \ u(0.5,y) = 200y.$$

(a) (10%) Partition the interval $[0, 0.5]$ into $n = 4$ equal parts. Please extend the finite difference method in Examination (1) to approximate the solution of above elliptic PDE and write down the associated linear system with matrix form $Ax = b$ (Do not need to solve this linear system).

(b) (20%) Write down the algorithm of Gauss-Seidel iterative method for solving general linear system $Gx = c$ with n-by-n nonsingular matrix G and give a simple implementation of this algorithm by MATLAB, C or Fortran.

(c) (15%) Can you guarantee that the Gauss-Seidel method for solving linear system $Ax = b$ in (a) converges? Why?

(d) (10%) Which iterative method for solving linear system $Ax = b$ in (a) is more efficienct? Why?

(3) (15%) For any $x_0 \in [0, 2\pi]$, the sequence $\{x_n\}$ is defined by

$$x_n = \pi + 0.5 \sin(x_{n-1}/2), \ n \geq 1.$$

The sequence $\{x_n\}$ converges or diverge. Why?

(4) (15%) Show that the polynomial interpolating the following data has degree three.

x	-2	-1	0	1	2	3
$f(x)$	1	4	11	16	13	-4