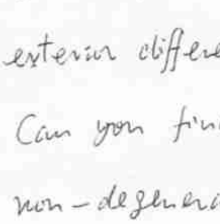
博士班資格考－幾何

92.2月

1. $\mathbb{R}^4 = \{(x,y,z,w) \mid -\infty < x,y,z,w < \infty\}$. exterior differential 2-form $\Omega = \sin x \, dx \wedge dy + \sin x \, dz \wedge dw + \cos x \, dy \wedge dz - \cos x \, dw \wedge dx$. Is Ω a symplectic 2-form? Can you find an exterior differential 2-form which is closed but degenerate? Can you find an exterior differential 2-form which is non-degenerate but not closed? (25/100)

2. $\mathbb{R}^3 - \{0\} = \{(x,y,z) \neq (0,0,0)\}$. Is this space simply-connected? $\Omega = (x^2+y^2+z^2)^{-\frac{3}{2}} (x \, dy \wedge dz + y \, dz \wedge dx + z \, dx \wedge dy)$ Is Ω an exact differential 2-form? If not, explain why not; if yes, find an exterior 1-form ω so that $\Omega = d\omega$ (25/100)

3. $M = \{ z = x^2 - y^2 \}$ is a surface in $\mathbb{R}^3$. $P = (1,0,1)$



$Q = (0,1,-1)$, $R = (-1,0,1)$, $S = (0,-1,-1)$

Ω = domain bounded by $\overline{PQ}, \overline{QR}, \overline{RS}, \overline{SP}$.

$\vec{n}$ = unit normal vector, surface integral $\iint_\Omega \vec{n} \, dS = (?,?,?)$ If $\tilde{\Omega}$ is another domain having the same boundary as Ω, $\vec{\nu}$ = its unit normal vector. Can you prove $\iint_{\tilde{\Omega}} \vec{\nu} \, dS = \iint_\Omega \vec{n} \, dS$? (25/100)

4. Can you find a Kähler manifold which is sequentially compact? I.e, each infinite sequence of points in this manifold has a convergent sub-sequence (25/100)