博士班 資格考 - 設計理論

Definitions:

- Let G be a group of order v. A (v, k, λ) difference set in G is a set D of k elements such that the "difference" $x \cdot y^{-1}$ with $x, y \in D$ contains every non-identy exactly λ times.

- A $t - (v, k, \lambda)$ design is a set D of k subsets of a given set of v points, such that every set of t points is contained in exactly λ members of D.

Problems (20 % each)

1. Construct a finite field $F = GF(3^3)$ of order 27 .

2. Find a $(27, 13, 6)$ difference set (hint: using the set of quadratic residues in $GF(3^3)$).

3. Using GF(27) to construct the projective 2-design 2-$(13, 4, 1)$.

4. Using Singer's theorem to construct a $(13, 4, 1)$ difference set.

5. Prove or disprove the existence of the following difference sets. (hint: you may use the multiplier theorem to construct the difference set)

   (a) $(7, 4, 2)$ difference set,

   (b) $(15, 7, 3)$ difference set,

   (c) $(21, 5, 1)$ difference set,

   (d) $(22, 7, 2)$ difference set,

   (e) $(73, 9, 1)$ difference set.

6. Find two non-isomorphic 2-designs with same same set of parameters (v, k, λ).

7. Find an infinite sequnece of $3 - (v_n, k_n, \lambda_n)$ designs with $\lambda_n = 1$

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