NCTU Department of Applied Mathematics

Qualifying Examination in Discrete Mathematics

for the Ph. D. Program September 2017

<u>**Note**</u>: The proofs and statements must be detailed. When you quote some theorems, please prove them.

1. Prove P. Hall's marriage theorem by two different methods. (20%)

2. Let n be a positive integer, U be the set of all trees with the vertex set $\{1,2,\ldots,n\}$, and $S=\{T\in U$: the degree of n in T is $1\}$. Find the cardinality of S. (20%)

3. Let t be a positive integer, M_t be the set of all 2×t matrices over $\{1,2,\ldots,t\}$, and $C=\{A\in M_t : A=[a_{ij}]$ with $a_{1i}\neq a_{2i}$ for $i = 1,2,\ldots,t$ and $\{a_{11},a_{12},\ldots,a_{1t}\}=\{a_{21},a_{22},\ldots,a_{2t}\}=\{1,2,\ldots,t\}$. Find the cardinality of C. (20%)

4. Let n be an odd integer with $n > 0$. Prove that if n is not divided by 3 then there are 4 mutually orthogonal Latin squares of order n. (20%)

5. True or False. (If the statement is true, prove it; if it is false, give a counterexample) (10%×2)

(a) Let G be a simple graph. If the chromatic number of G ≥ 5 then G has a subgraph which is isomorphic to a complete graph of order 3.

(b) Let n be an integer with $n \geq 3$, G be a simple graph of order n, and the maximum degree of G be k. Then the chromatic number of G is k+1 if and only if G is isomorphic to a complete graph of order n or a cycle of order n.