2025 NCTS Undergraduate Summer Research Program

Research Topic : The mathematical analysis of the kinetic theory

Advisors : I-Kun Chen / ikunchen@gmail.com
Chun-Hsiung Hsia / willhsia@math.ntu.edu.tw

TAs : Jin Zhi Phoong / d11221002@ntu.edu.tw
Kai Li Wang / r13221012@ntu.edu.tw

Background and Motivation

The existence theory and the regularity theory are two important issues in the study of PDE problems. To establish the existence theory, one can resorts to :

(1) the compactness of the solution operator to apply fixed point arguments or contraction mapping arguments.
(2) the existence theory by functional analysis, for example : the Fredholm alternative, the method of continuity, semigroup theory, .. etc.

The regularity theory seems to be the next issue after one obtains the existence theory. In fact, the regularity can help to establish the compactness of solution operators, and therefore helps to establish the existence theory.

In this program, we have prepared the materials for students to study so that they can get a good understanding of the aforementioned theory and try to apply this framework to investigate the Boltzmann equations.

Plan

Stage 1 : Book reading, May 1st – June 30th , 2025.
The participants shall read the following prelims by themselves.
a. Ch 1 – Ch 3 of the book by Glassey [1] to learn the basic properties of the linear theory of the Boltzmann equation.
b. Reference [2] to have a basic idea of the compactness in fractional Sobolev spaces.
c. Ch. 6 of [3] to learn the Fredholm Alternative theory.

Stage 2. Paper reading, July 1st – July 30th
In the first four weeks of the USRP, the two TAs will lead daily seminars on the Ref [4,5,6,7]. The advisors will give a few lectures and explain more about the key points and the global pictures of the research goals.

Stage 3. Problem solving, Aug 1st – Aug 15th
In this period of time, the advisors will give open problems to students to work. But, we do not expect the participants can come up with resolutions during the USRP period. This is just an initiation.

Eligibility

This is a tough research training. The students should have already learned the (1) the undergraduate level analysis and (2) undergraduate level PDEs. We only welcome the participants who are seriously interested in the research of advanced analysis.

Reference

[1] The Cauchy Problem in Kinetic Theory, Robert Glassey.
[2] Hitchhiker's guide to the fractional Sobolev spaces, Eleonora Di Nezza, Giampiero Palatucci, Enrico Valdinoci.
[3] Functional Analysis, Sobolev Spaces and Partial Differential Equations, Haim Brezis.
[4] On the existence and regularity of weakly nonlinear stationary Boltzmann equations : a Fredholm alternative approach, I-Kun Chen, Chun-Hsiung Hsia, Daisuke Kawagoe. https://arxiv.org/abs/2501.02419
[5] A revisit of the velocity averaging lemma: On the regularity of stationary Boltzmann equation in a bounded convex domain, I-Kun Chen, Ping-Han Chuang, Chun-Hsiung Hsia, Jhe-Kuan Su; J. Stat. Phys. 189 (2022), no.2, Paper 17, 43 pp.
[6] Mouhot, Cl´ement : Explicit coercivity estimates for the linearized Boltzmann and Landau operators, Comm. Partial Differential Equations 31 (2006), no. 7-9, 1321–1348.
[7] MIXTURE LEMMA, HUNG-WEN KUO, TAI-PING LIU AND SE EUN NOH, Bulletin of the Institute of Mathematics, Academia Sinica (New Series) Vol. 5 (2010), No. 1, pp. 1-10.